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OPTICS OF THE AIR.*

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INTRODUCTION—CLASSIFICATION.

MANY curious and beautiful phenomena, of which the mirage, the rainbow, the halo, the azured sky, and the twilight glow, are some of the more conspicuous, are due to the optical properties of the air and the foreign substances suspended in or falling through it. All, or nearly all, of them have been the objects of innumerable observations and many careful studies, the results of which, fortunately, have been summarized and discussed by various authors. The most extensive discussion, however, of this subject is by Pernter and Exner, whose work, "Meteorologische Optik," therefore, will be largely, but by no means exclusively, drawn upon for the material of this section.

When one's chief or only purpose in discussing the optics of the air is to describe the phenomena seen, it is convenient to divide them into such general classes as mirages, rainbows, halos, coronas, etc. For explanatory purposes it is more convenient, perhaps, to group them according to (*a*) their objective or material causes, namely: atmosphere, raindrops, water droplets, ice crystals, etc.; or (*b*) their physical causes, such as reflection, refraction, diffraction, etc.

* Continued from page 509, Vol. 186, October, 1918.

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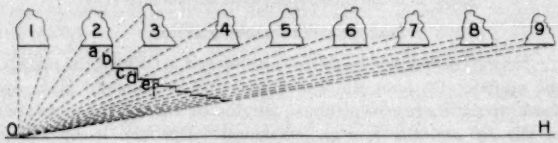
Each of the above classifications has its advantages and disadvantages. On the whole, however, the division according to physical causes seems best suited to the needs of an explanatory discussion and therefore is here adopted.

CHAPTER I.

PERSPECTIVE PHENOMENA.

Apparent Stair-step Ascent of Clouds.—The stair-step appearance of the echelon cloud (Fig. 103) is, perhaps, the simplest sky phenomenon due to perspective. The exact manner by which the stair-step or terrace illusion is brought about is shown by Fig. 123, in which *O* is the position of the observer, *H* his horizon, 1, 2, 3, etc., evenly spaced flat-bottomed cumuli of the same

FIG. 123.



Cloud echelon effect.

base elevation—flat-bottomed and of constant level because of the approximately uniform horizontal distribution of moisture.

Since the clouds are at a higher level than the observer, each successive cumulus, as the distance increases, is seen at a lower angle than its predecessor; and the dark bases of any two adjacent clouds appear to be connected with each other by the lighter side of the farther one. Besides, their general resemblance to stair-steps often leads one into the error of "seeing" the connection between any two adjacent bases to be at right angles to both. That is, starting with base *a*, the light side of cloud 3 appears as a vertical surface at *b*, and its base as a dark horizontal surface at *c*; the side and base of cloud 4 appear as the next vertical and horizontal surfaces, *d* and *e*, respectively, and so on for the other clouds; the whole effect merging into the appearance of a great stairway, consisting of the horizontal treads, *a*, *c*, *e*, etc., connected by the seemingly vertical risers, *b*, *d*, *f*, etc.

Apparent Arching of Cloud Bands.—Occasionally a narrow

cloud band is seen to stretch almost, if not entirely, from horizon to horizon, but although its course is practically horizontal and its direction often nearly straight, it usually appears arched. If even the nearest portion of the cloud still is far away, the apparent arching is slight. On the other hand, when the cloud is near the arching is great. The apparent curve is neither circular nor elliptical, but resembles rather a conchoid whose origin is at the observer and whose asymptote is his horizon.

The angle of elevation at which different segments of the cloud are seen clearly varies from a minimum for the more distant portions to a maximum for the nearest. Hence, the phenomenon in question, the apparent arching of the band along its nearest portions, is only an optical illusion, due entirely to the projection of the cloud (above the observer's level) onto the sky.

When several such bands or streaks occur in parallel, they appear to start from a common point at or beyond the horizon, to terminate, if long enough, in a similar opposite point, and progressively to arch and spread apart as they approach the observer's zenith. They thus form the perspective effect often called "Noah's Ark" or polar bands.

Apparent Divergence and Convergence of Crepuscular Rays (Sunbeams).—Everyone is familiar with the beautiful phenomenon of the "sun drawing water"—sunbeams that, finding their way through rifts in the clouds, are rendered luminous by the dust in their courses. Equally familiar and equally beautiful are also those streaks and bands of pearly lights (where the lower atmosphere is illuminated) and azure shadows (where only the upper atmosphere is illuminated) that often at twilight and occasionally at dawn radiate far out from the region of the sun, and at times even converge towards the opposite point of the horizon. These, too, are only beams of sunlight and shadow caused by broken clouds or irregular horizon.

All such crepuscular rays, whether their common origin, the sun, be below or above the horizon, seem first to diverge, while the few that cross the sky appear also to arch on the way and finally to converge towards the antisolar point.

Here, again, the facts are not as they seem, for the rays, all coming, as they do, from the sun, some 93,000,000 miles away, necessarily are practically parallel. Their apparent divergence, convergence, and arching are all illusions due to perspective, just

as are the apparent divergence, convergence, and arching of the rails on a long straight track.

Apparent Divergence of Auroral Streamers.—Anyone at all familiar with the appearance of auroral streamers will recall that at most localities they seem to radiate from some place far below the horizon. In reality they do diverge (or converge, if one prefers) slightly since they follow, approximately, the terrestrial lines of magnetic force. Indeed, their rate of convergence is about the same, on the average, as that of the geographic meridians at the same latitudes, and therefore far less than one would infer from their apparent courses. That is, their seeming rapid convergence is only another illusion due to perspective, just as is the apparent divergence of the crepuscular rays, as above explained.

Apparent Shape (Flat Vault) of the Sky.—To everyone the sky looks like a great blue dome, low and flattish, whose circular rim rests on the horizon and whose apex is directly overhead. So flat, indeed, does this dome appear to be that points on it estimated to lie half-way between the rim and apex generally have an elevation of but little more than 20°, instead of 45°, as they would if it seemed spherical.

That the rim of the sky dome should appear circular is obvious enough. It is simply because the horizon, where land and sky come together, itself is circular, except when conspicuously broken by hills or mountains.

To understand the other and more important feature, that is, why the dome looks so flat, consider (1) a sky filled from horizon to horizon with high cirrus clouds. These seem nearest overhead for the simple reason that that is just where they are nearest. As the horizon is approached, the clouds merge, through perspective, into a uniform gray cover that appears to rest on the land at the limit of vision, whether this limit be fixed by the curvature of the earth or by haze, and the whole cloud canopy may seem arched just as and for the same reason that cloud streaks and crepuscular rays seem arched, as above explained. But (2) even a thin cirro-stratus veil whose parts are well-nigh indistinguishable produces a similar effect, the nearest portions appearing nearest, largely because they are the most clearly seen. Similarly, when there are no clouds the sky overhead also appears nearest because it is clearest; and that unconscious inference,

based on endless experience, is correct—it is clearest because nearest. As the eye approaches the horizon, the increasing haze produces the impression of greater distance; and this impression is entirely correct, for the blue sky seen in any such direction is farther away than the sky overhead. In short, the spring of a cloudless sky dome is "seen" to rest on the distant horizon and its ceiling to come closer and closer, in proportion to increasing clearness, as the zenith is approached. The shape, then, of this dome should not always appear the same, and it does not—not the same on a clear night, for instance, as on a clear day.

Impressions, therefore, of the "shape" of the sky are, perhaps, not so erroneous as sometimes they are said to be. Indeed, they usually conform surprisingly well to the actual facts.

Change, with Elevation, of Apparent Size of Sun and Moon.—One of the most familiar, as also one of the most puzzling, of optical illusions is the change between the apparent sizes of the full moon, say, or of the sun, at rising or setting, and at or near culmination. It is, however, only a phenomenon of perspective.

Since the solid angle subtended at any place on the earth by the moon, as also that subtended by the sun, is sensibly constant throughout its course from rising to setting, it follows that its projection, and, therefore, its apparent size, must be relatively large, or small, as the place of projection (sky dome) is comparatively far away or nearby. But, as already explained, the sky dome, against which all celestial objects are projected and along which they therefore appear to move, seems to be farther away, and is farther away, near the horizon than at places of considerable elevation. Hence the moon and the sun must look much larger when near the horizon than when far up in the heavens, and the fact that they do so look, is, as stated, merely a phenomenon of perspective.

The familiar fact that the moon appears of one size to one person and a different size to another clearly is also due to perspective. The one who judges it large imagines his comparison object to be at a greater distance than does the one who judges it small. Why two people, however, should differ so widely in their reference or comparison distances, as obviously they often do, is by no means clear.

Change, with Elevation, of Apparent Distance Between Neighboring Stars.—The generally recognized fact that the dis-

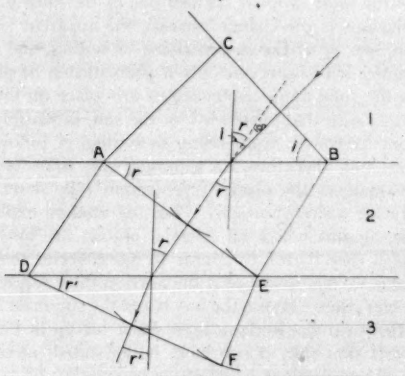
tance between neighboring stars appears much greater when they are near the horizon than when well up is also a phenomenon of perspective. Its explanation is identical with that of the change, under similar circumstances, of the apparent diameter of the moon, and therefore need not be given in further detail.

CHAPTER II.

REFRACTION PHENOMENA: ATMOSPHERIC REFRACTION

Astronomical Refraction.—It is well known that because of astronomical refraction the zenith distance of a star, or other celestial object, is greater than it seems, except when zero, to an extent that increases with that distance. To understand this im-

FIG. 124.



Refraction of light on change of media.

portant phenomenon, it is necessary to recall two experimental facts: (*a*) that in any homogeneous medium light travels in sensibly straight lines, and (*b*) that its velocity (velocity pertaining to any given wave frequency) differs from medium to medium.

Let, then, the parallel lines *AB* and *DE* (Fig. 124) be the intersections of the boundaries between three homogeneous media, 1, 2, 3, by a plane normal thereto and to the wave front, *BC*. Let the velocities in these media of a given monochromatic light be

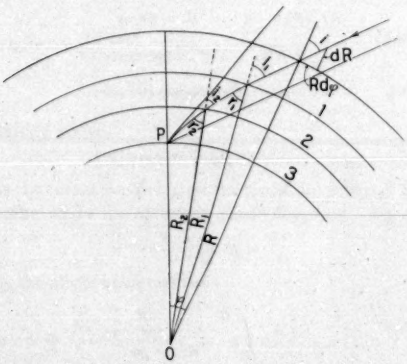
v_1, v_2, and v_3, respectively. Hence, when the light disturbance at C has travelled the distance CA in the first medium, that at B will have gone the distance BE in the second, where $CA/BE = v_1/v_2$, and AE will be the new wave front. Similarly, DF will be the wave front in the third medium, and so on for any additional media that may be traversed.

If i is the angle between the normal to the interface, AB, and the direction of the light, both in medium 1, and r the corresponding angle in medium 2, then, as is obvious from the figure,

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}, \text{ or } \sin i = \frac{v_1}{v_2} \sin r.$$

Similarly, $\sin r = \frac{v_2}{v_3} \sin r'$. Hence, $\sin i = \frac{v_1}{v_3} \sin r'$. That is, the total change in direction of the light depends solely on its velocities in the first and final media, respectively, and the initial angle of incidence. The optical densities of the intermediate

FIG. 125.



Path of light through the atmosphere.

layers may abruptly change by large amounts, as indicated, and thus cause the light to follow a perceptibly broken course, as from air to water, for instance; or they may change so gradually that the path is a smooth curve, even to the closest observation.

Since the ratio of the velocity of light in space to its velocity in any given gas, or definite mixture of gases (the refractive index of that medium), increases directly with density, it follows that all rays of light that cross the atmospheric shell, except those that enter it normally, must follow continuously curved paths, somewhat as shown to an exaggerated extent in Fig. 125.

To determine the shape of such a curve through the atmosphere, let ϕ (Fig. 125) be the angle between the radii from the centre of the earth at the place of observation and any other point along the course of a refracted ray.

As before,

$$\mu_2 \sin r_1 = \mu_1 \sin i_1$$

in which μ_1 and μ_2 are the refractive indices (with reference to space) of media, or layers, 1 and 2, respectively, i_1 the angle of incidence and r_1 the angle of refraction at the interface between these media or layers. But, corresponding to the radii R_1 and R_2,

$$\frac{\sin i_2}{\sin r_1} = \frac{R_1}{R_2}$$

Hence,

$$R_1 \mu_1 \sin i_1 = R_2 \mu_2 \sin i_2,$$

or, in general,

$$R \mu \sin i = C, \text{ a constant.}$$

Further,

$$\frac{dR}{Rd\phi} = \cot i.$$

But $\cot i = \dfrac{\cos i}{\sin i} = \sqrt{\dfrac{1 - \sin^2 i}{\sin^2 i}} = \sqrt{\dfrac{\mu^2 R^2}{C^2} - 1}$

Hence,

$$\frac{dR}{d\phi} = R \sqrt{\frac{\mu^2 R}{C^2} - 1},$$

and

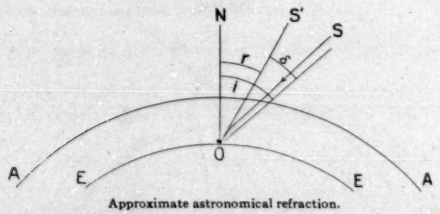
$$\phi = \int_{R_0}^{R} \frac{dR}{R \sqrt{\dfrac{\mu^2 R^2}{C^2} - 1}}$$

Clearly, then, the value of ϕ corresponding to a definite value of R, or the value of R appropriate to a definite value of ϕ, depends upon the relation of μ to R, or, very nearly, the relation of

the density of the atmosphere at any point to the altitude of that point. Hence, refraction curves may be drawn for different angles of incidence, or, if preferred, for different apparent altitudes, according to any assumed distribution of atmospheric density—a distribution fairly well known.

The approximate value of astronomical refraction, that is, its value generally to within one second of arc, through all zenith distances up to at least 60°, may easily be obtained as follows: Assume the atmosphere to be flat, as it nearly is, over the restricted area through which stars may be seen whose zenith distances are within 60°, or thereabouts. Let O (Fig. 126) be the position of the observer, S the true position of a star and S' its apparent position.

FIG. 126.



Approximate astronomical refraction.

As explained above,

$$\sin i = \mu \sin r$$

in which μ is the refractive index of the air at the point of observation, i the actual and r the apparent zenith distance. But

$$i = r + \delta$$

in which δ is the angle of deviation.

Hence,

$$\sin (r + \delta) = \sin r \cos \delta + \cos r \sin \delta = \mu \sin r$$

When the angle of incidence is 60°, or less, δ is always very small, and

$$\sin \delta = (\mu - 1) \tan r, \text{ nearly.}$$

Expressed in seconds of arc this gives

$$\delta'' = 206265'' (\mu - 1) \tan r$$

in which the numerical coefficient is the approximate number of seconds in a radian.

For dry air at 0° C. and 760 mm. pressure, the average value of μ is about 1.000293.

Hence, also

$$\delta'' = 60''.4 \tan r.$$

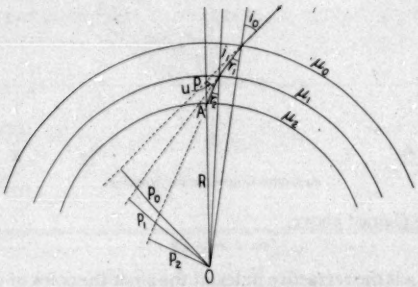
But for gases $\mu - 1 = K\rho$, very closely, in which K is a constant and ρ the density.

Hence, finally,

$$\delta'' = \frac{21''.7\ B}{T} \tan r,$$

in which B is the height of the barometer in millimetres, and T the absolute temperature in degrees C.

FIG. 127.



Astronomical refraction (Lord Rayleigh).

As a matter of fact, the atmospheric shell is not plane, even over small areas, but slightly curved, and therefore the complete formula for astronomical refraction, such as is needed for the construction of tables to be used in the most accurate measurements of star positions, is rather complicated. Probably the briefest and simplest derivation of a formula adequate for all zenith distances to at least 75° is due to Lord Raleigh,[167] and is essentially as follows:

Let p_0, p_1, p_2, etc., be the normals from the centre of the earth onto the tangents of a ray path through the atmosphere at the

[167] *Phil. Mag.*, 36, p. 141, 1893.

points where the refractive indices are μ_0, μ_1, μ_2, etc., respectively. Let i_0, i_1, i_2, etc., be the angles of incidence, and r_1, r_2, etc., the corresponding angles of refraction. Then (see Fig. 127),

$$\frac{\mu_0}{\mu_1} = \frac{\sin r_1}{\sin i_0} = \frac{p_1}{p_0},$$

$$\frac{\mu_1}{\mu_2} = \frac{\sin r_2}{\sin i_1} = \frac{p_2}{p_1}, \text{ etc.}$$

Hence,

$$\mu p = \text{constant.}$$

Let the tangent to the ray path, where it enters the atmosphere, meet the vertical at the distance, C, above the point of observation, A; let μ_s be the refractive index at A; θ the apparent zenith distance; $\delta\theta$ the total refraction; and R the radius of the earth. Then, since the refractive index of space is 1,

$$\mu_s p_s = p_0, \text{ or } \mu_s R \sin\theta = (R + C) \sin(\theta + \delta\theta) \ldots\ldots \text{(A)}$$

Obviously, the refraction, $\delta\theta$, could be determined from this equation directly if the value of C were known. But

$$C = \frac{u}{\sin\theta} \ldots\ldots\ldots\ldots\ldots\ldots \text{(B)}$$

in which u, the total linear deviation of the ray, may be substituted by known terms. Hence, C and, therefore, $\delta\theta$ are determinable.

To determine u, let α be the angle which the ray makes with the direction of most rapid increase of index of refraction (at the surface α equals θ); z the vertical coördinate; and v the velocity of light. Now consider a wave front moving through the atmosphere in any direction except vertical. The portion in the higher or thinner air will move faster than that in the denser air and the path will be curved. If ρ is the radius of curvature and $d\rho$ is regarded as positive when measured towards the centre, then, as is obvious from Fig. 128,

$$\frac{d\rho}{\rho} = -\frac{dv}{v}$$

Also, since the refractive index, μ, is inversely proportional to the velocity, v,

$$\frac{d\rho}{\rho} = \frac{d\mu}{\mu}$$

Hence, calling the path s, and since

$$\frac{d\mu}{d\rho} = \frac{d\mu}{dz}\sin\alpha,$$

$$\frac{1}{\rho} = \frac{d\log\mu}{d\rho} = \frac{d\log\mu}{dz}\sin\alpha = \frac{d\log\mu}{ds}\tan\alpha = \frac{d^2u}{ds^2}$$

To a close approximation,

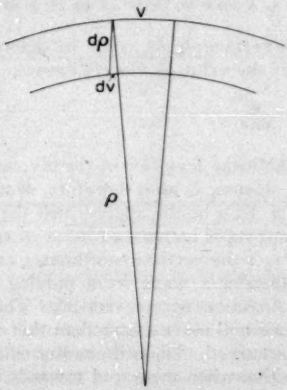
$$\frac{du}{ds} = \int\tan\alpha\ d\log\mu = \tan\alpha\ (\mu - 1) + a,$$

in which a is the constant of integration and $\mu - 1$ is substituted for $\log_e\mu$, μ being but little greater than 1, and

$$u = \tan\alpha\int(\mu - 1)\,ds + as + b,$$

$$= \frac{\sin\alpha}{\cos^2\alpha}\int(\mu - 1)\,dz + as + b \ldots\ldots (C)$$

FIG. 128.



Curvature of a light path in the air.

But $\mu - 1$ is directly proportional to the density. Hence, if the height, h, in "the homogeneous atmosphere" be such that air below it is the same as below z in the actual atmosphere, and if the origin be taken at the surface where $\alpha = \theta$, and $\mu = \mu_0$

$$u = \frac{\sin\theta}{\cos^2\theta}\int_0^h(\mu - 1)\,dz = \frac{\sin\theta}{\cos^2\theta}(\mu_0 - 1)\,h.$$

For the limit of the atmosphere, and any point beyond, $h = H$, the height of the homogeneous atmosphere, or about 7.990×10 cm.

For stars, therefore, viewed from the surface of the earth,

$$u = (\mu_s - 1)\, H \frac{\sin \theta}{\cos^2 \theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (D)$$

Substituting this value of u in (B) we get

$$C = \frac{(\mu_s - 1)\, H}{\cos^2 \theta}.$$

Hence, substituting in (A),

$$\mu_s R \sin \theta = \left\{ R + \frac{(\mu_s - 1) H}{\cos^2 \theta} \right\} \left\{ \theta + \delta\theta \right\}.$$

With this equation it is only a matter of arithmetic to compute a table of corrections, that is, values of $\delta\theta$ for every value of θ and μ_s; θ being the apparent or observed zenith distance, and μ_s the current refraction at the surface, given by the equation

$$\mu_s = \frac{273 \mu_0 B}{760\, T}$$

in which B is the height of the barometer in millimetres, T the absolute temperature in degrees C., and μ_0 atmospheric refraction at 0° C. and 760 mm. pressure.

Since $\delta\theta$ is small, we have from (A), to a close approximation,

$$\begin{aligned} \delta\theta = \sin \delta\theta &= \frac{\mu_s \tan\theta}{1 + C/R} - \tan\theta \cos \delta\theta \\ &= \tan\theta \left(\frac{\mu_s}{1 + C/R} - \sqrt{1 - \sin^2 \delta\theta} \right) \\ &= \tan\theta \left\{ \frac{\mu_s}{1 + C/R} - 1 + \tfrac{1}{2} (\delta\theta)^2 \right\} \\ &= \tan\theta \left\{ \mu_s - \frac{\mu_s C}{R} - 1 + \tfrac{1}{2} (\delta\theta)^2 \right\}, \text{ nearly.} \end{aligned}$$

But, from the laws of refraction,

$$\mu_s \sin\theta = \sin (\theta + \delta\theta)$$

from which

$$\delta\theta = (\mu_s - 1) \tan\theta, \text{ nearly.}$$

Hence,

$$\delta\theta = (\mu_s - 1) \tan\theta \left\{ 1 - \frac{\mu_s C}{(\mu_s - 1) R} + \tfrac{1}{2} (\mu_s - 1) \tan^2 \theta \right\}, \text{ approximately.}$$

Substituting for C its value and noting that

$$\frac{1}{\cos^2\theta} = 1 + \tan^2\theta$$

this equation reduces to

$$\delta\theta = (\mu_s - 1)\left(1 - \frac{\mu_s H}{R}\right)\tan\theta - (\mu_s - 1)\left(\frac{\mu_s H}{R} - \frac{\mu_s - 1}{2}\right)\tan^2\theta$$

$$= (\mu_s - 1)\left(1 - \frac{H}{R}\right)\tan\theta - (\mu_s - 1)\left(\frac{H}{R} - \frac{\mu_s - 1}{2}\right)\tan^2\theta, \text{ nearly.}$$

If $H = 7.990 \times 10^5$ cm.; $R = 6.3709 \times 10^8$ cm.; and $\mu_s - 1 = .0002927$, all closely approximate values, then,

$$\delta\theta = 60''.29\tan\theta - 0''.06688\tan^2\theta.$$

This is Lord Rayleigh's final equation, and it appears to be exceedingly accurate for all values of θ up to at least 75°, or as far, perhaps, as irregular surface densities generally allow any refraction formula to be used with confidence.

Since the index of refraction varies from color to color, it follows that star images are drawn out into vertical spectra. The amount of this effect, however, is small. Thus the difference between the refractions of red and blue-green is only about one one-hundredth of the total refraction of yellow (D) light. Hence, the approximate angular distance between the red and blue-green images of a star at the zenith distances, 30°, 45°, 60° and 75° are 0″.35, 0″.60, 1″.04, and 2″.24, respectively. Possibly this may account, in part at least, for the fact that stellar declinations, as determined from the northern and southern hemispheres, respectively, are not quite the same.

Scintillation or Twinkling and Unsteadiness of Stars.—The scintillation or twinkling of stars, that is, their rapid changes in brightness and occasionally also in color, especially when near the eastern or western horizon, is a well-known, and now well-understood, phenomenon that for many centuries, certainly since the days of Aristotle (384–322 B.C.), who noted the fact that the fixed stars twinkle while the planets shine with comparatively steady lights, has been observed, investigated, and discussed. The most systematic and complete observations, however, of scintillation are those made by Respighi [168] with a spectroscope

[168] Assoc. Française pour l'Avancement des Sciences, I, 1872, p. 148.

during the years 1868–1869, and summed up substantially as follows:

(1) In the spectra of stars near the horizon more or less broad and distinct dark and bright bands sweep with greater or less velocity from the red to the violet and from the violet to the red or oscillate from the one to the other; and this whatever the direction of the spectra from the horizontal to the vertical.

(2) When the conditions of the atmosphere are normal the dark and bright bands of western stars travel regularly from the red to the violet, and of eastern from the violet to the red; while in the neighborhood of the meridian they usually oscillate from the one color to the other, or even are limited to a portion of the spectrum.

(3) On examining the horizontal spectrum of a star sensibly parallel dark and bright bands more or less inclined to the axis (transverse) of the spectrum are seen passing from the red to the violet or reversely, according as the star is in the west or east.

(4) The inclination of the bands, or angle between them and the axis (transverse) of the spectrum, depends upon the altitude of the star; increasing rapidly from 0° at the horizon to 90° at an elevation of 30° to 40°, where they, therefore, are longitudinal.

(5) The inclination of the bands, reckoned from above, is towards the violet end of the spectrum.

(6) The bands, most distinct at the horizon, become less conspicuous with increase of elevation. Above 40° the longitudinal bands reduce to mere shaded streaks and often can be seen in the spectrum only as slight general changes of brightness.

(7) With increase of elevation the movement of the bands becomes more rapid and less regular.

(8) On turning the spectrum from the horizontal the inclination of the bands to the transversal continuously diminishes until it becomes zero, when the spectrum is nearly vertical. They also become less distinct, but continue always to move in the same direction.

(9) The bright bands are less frequent and more irregular than the dark, and are well-defined only in the spectra of stars near the horizon.

(10) In the midst of this general and violent movement of light and shade over the spectra of stars, the Fraunhofer lines peculiar to the light of each star remain quiescent or are subject to only very slight oscillations.

(11) When the atmospheric conditions are abnormal, the bands are fainter and more irregular in form and movement.

(12) When the wind is strong, the bands usually are quite faint and ill-defined; the spectra even of stars near the horizon showing mere changes of brightness.

(13) Good definition and regular movement of the bands appear to indicate the continuance of fair weather, while varying definition and irregular motion seem to imply a probable change.

These observations show that the dark bands are due to temporary deflection of light from the object glass by irregularities in the density of the atmosphere. For stars near the horizon, the linear separation of the rays of different color is so great as they pass through the atmosphere to the observer that successive portions of the spectrum may be deflected from or concentrated onto (light deficiencies in a transparent medium must be balanced by light concentrations, and *vice versa*), the telescope or eye. Hence, the progression of bright and dark bands along the spectra of low altitude stars, and their rapid change of color to the unaided eye.

Further, since the path of the more refrangible light necessarily lies above that of the less refrangible (Montigny's principle) it follows that an atmospheric irregularity travelling or

rotating with the earth would affect the different-colored rays from stars in the west in the order of red, green, blue, violet, and rays from stars in the east in the reverse order. If, then, the separation of the extreme rays is large in comparison to the effective dimension of the air irregularity, the resulting band will, at any given instant, cover only a portion of the spectrum. But the approximate amount of this separation is readily obtained from equation (D), page 445. Thus, for the limit of the atmosphere,

$$du = d\mu_s H \frac{\sin\theta}{\cos^2\theta}$$

Hence, at the zenith distance, 80°, the red and violet rays simultaneously received by the observer from the same star will be separated at the limit of the atmosphere (assuming the dispersion between these rays to be one-fiftieth the refraction of yellow light) by about 156 centimetres, and proportionately for levels below which definite fractions of the total mass of the atmosphere lie. For the zenith distance, 40°, however, the corresponding separation at the limit of the atmosphere is only about 5 centimetres, and for 20° about 2 centimetres. Inequalities in the atmosphere may, therefore, interfere with only a portion at a time of the spectrum of a star near the horizon and thus produce the phenomenon of a travelling band, while in the case of a star whose zenith distance is 40°, or less, the interference will include nearly or quite the entire spectrum, and thus produce mere changes of brightness.

It is generally stated that the direction of travel of the bands during fine weather, red to violet for stars in the west, violet to red for stars in the east, and irregularly or simultaneously over the entire spectrum for stars near the meridian, is directly dependent upon the west to east rotation of the earth. It is correctly stated (on assumption of a stationary atmosphere) that this rotation would cause an atmospheric irregularity to affect the red rays first and the violet last, violet first and red last, and all rays more or less simultaneously, of stars in the west, east, and near the meridian, respectively. But the order would be the same if the earth were at rest and the air travelling from west to east. As a matter of fact, over most of the earth outside the tropics the west to east angular velocity of the general winds, as seen by the observer, is several times that of the earth. Hence,

the rate at which the disturbance drifts across the line of sight presumably depends much more on the direction of the prevailing winds than upon the rotation of the earth. Indeed, in tropical regions, where the prevailing winds are from easterly points, the usual direction of travel of the bands probably (if the above reasoning is correct) is reversed.

The disappearance of distinct bands with high winds is due, of course, to the more complete mixing of the atmosphere at such times.

In the same general way, atmospheric inequalities produce "unsteadiness," or rapid changes in the apparent positions of stars as seen in a telescope. In reality, this is a telescopic form of scintillation which, because never amounting to more than a very few seconds of arc, the unaided eye cannot detect. On the other hand, the great changes in brightness and color so conspicuous to the naked eye are scarcely if at all noticeable in a large telescope. This is because the object glass is so large that, in general, light deflected from one portion of it is caught in another.

Scintillation of the Planets, Sun, and Moon.—It is commonly stated that the planets do not scintillate—that the light from the several portions of their disks follow such different paths through the atmosphere that not all nor even any large portion of it can be affected at any one time. It is true that because of their sensible disks the scintillation of planets is much less than that of fixed stars, but under favorable circumstances their scintillation is quite perceptible. Even the rims of the sun and the moon boil or "scintillate" while, of course, any fine marking on either or on a planet is quite as unsteady as the image of a fixed star.

Nature of Irregularities.—It is well known that the atmosphere, generally, is so stratified that with increase of elevation many more or less abrupt changes occur in temperature, composition, density, and, therefore, refrangibility. As such layers glide over each other, billows are formed, and the adjacent layers thereby corrugated. The several layers frequently also heat unequally, largely because of disproportionate vapor contents, and thereby develop, both day and night, and at various levels, innumerable vertical convections; each moving mass differing, of course, in density from the surrounding air, and by the chang-

ing velocity being drawn out into dissolving filaments. Optically, therefore, the atmosphere is so heterogeneous that a sufficiently bright star shining through it would produce on the earth a somewhat streaky pattern of light and shade.

Shadow Bands.—A striking proof of the optical streakiness of the atmosphere is seen in the well-known shadow bands that at the time of a total solar eclipse appear immediately before the second, and after the third, contact.

Terrestrial Scintillation.—A bright terrestrial light of small size, such as an open electric arc, scintillates when seen at a great distance, quite as distinctly as do the stars and for substantially the same reason, that is, optical inequalities due to constant and innumerable vertical convections and conflicting winds.

Shimmering.—The tremulous appearance of objects, the common phenomenon of shimmering, seen through the atmosphere immediately over any heated surface, is another manifestation of atmospheric refraction, and is due to the innumerable fibrous convections that always occur over such an area.

Optical Haze.—The frequent indistinctness of distant objects on warm days when the atmosphere is comparatively free from dust, and ascribed to optical haze, is due to the same thing, namely, optical heterogeneity of the atmosphere, that causes that unsteadiness or dancing of star images that so often interferes with the positional and other exact work of the astronomer. Both are but provoking manifestations of atmospheric refraction.

Times of Rising and Setting of Sun, Moon, and Stars.—An interesting and important result of astronomical refraction is the fact that the sun, moon, and stars rise earlier and set later than they otherwise would. For places at sea level the amount of elevation of celestial objects on the horizon averages about 35′, and therefore the entire solar and lunar disks may be seen before (on rising) and after (on setting) even their upper levels would have appeared, in the first case, or disappeared, in the second, if there had been no refraction. This difference in time of rising, or setting, depends on the angle or inclination, α, of the path to the horizon. In general, it is given by the equation,

$$t = 140^s \csc\alpha, \text{ about.}$$

The minimum time, therefore, occurs when the path is normal to the horizon and is about $2^m\ 20^s$, while the maximum, which

obviously occurs at the poles, is infinite in the case of stars that just clear the horizon, and, for the sun, about a day and a half, the time required near equinox for the solar declination to change by 35′.

Green Flash.—As the upper limb of the sun disappears in a clear sky below a distant horizon its last star-like point often is seen to change rapidly from pale yellow or orange to green and finally blue, or, at least, a bluish-green. The vividness of the green, when the sky is exceptionally clear, together with its almost instant appearance, has given rise to the name "green flash" for this phenomenon The same gamut of colors, only in reverse order, occasionally is seen at sunrise.

The entire phenomenon has been described by some as merely a complementary after-image effect, that is, the sensation of its complementary color that frequently follows the sudden removal of a bright light. This explanation, however, cannot account for the reverse order of the colors as seen at sunrise. Neither does it account for the twinkling of the "flash" close observation now and then reveals, nor for the fact that when the sun is especially red the "flash" is never seen.

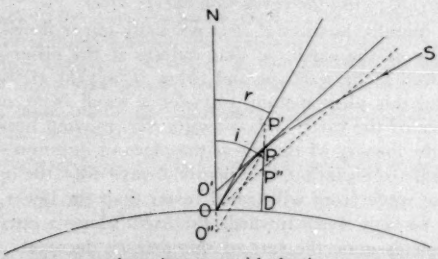
It is not, indeed, a physiological effect, but only the inevitable result of atmospheric refraction, by virtue of which as a celestial object sinks below the horizon its light must disappear in the order of refrangibility; the red first, being least refrangible, then the green, and finally the blue or most refrangible. Violet need not be considered, since only a comparatively small portion of it can penetrate so far through the atmosphere.

It may properly be asked, then, why these color changes do not apply to the whole solar disk. The answer is, because the angular dispersion, due to the refraction of the atmosphere, between the several colors is very small—between red and green, for instance, only about 20″, even when the object is on the horizon, so that any color from a given point on the sun is reinforced by its complementary color, thus giving white from a closely neighboring point. Hence, color phenomena can appear only when there are no such neighboring points, or when only a minute portion of the disk is above the horizon. It must further be noted that color effects, due to the general refraction of the atmosphere, occur only when the source (brilliant point) is on the horizon. Stars above the horizon are not permanently drawn out

into rainbow bands, and that for the simple reason that the red light by one route is supplemented by the green, blue, etc., by others and the whole blended into white, yellow, or whatever the real color of the star may be. This multiplicity of routes and consequent blending of color, is not possible for rays of light from objects just sinking below or rising above the horizon, and, therefore, under such circumstances, they pass through a series of color changes.

Terrestrial Refraction.—The curving of rays of light is not confined to those that come from some celestial object, but applies also to those that pass between any points within the atmosphere, whether at the same or different levels. This latter

FIG. 129.



Approximate terrestrial refraction.

phenomenon, known as terrestrial refraction, causes all objects on the earth or in the atmosphere to appear to be at greater altitudes than they actually are, except when the surface air is so strongly heated as to cause an *increase* of density with elevation and thus produce the inferior mirage, described below.

Terrestrial refraction is also a matter of great importance, especially to the geodesist, and its complete analysis, from which practical tables may be constructed, essentially the same as that of astronomical refraction. It will be instructive, however, to consider a few graphical corrections of apparent elevations.

Let ON (Fig. 129) be normal to the surface, OD, of the earth, and let P be observed from O. Obviously P, whose horizontal distance, OD, may be supposed known, will seem to be at P' and thus its apparent altitude greater than the actual by the distance

PP'. From the angle r, the density of the air at the observer's position, O, and the approximate density at P (known from the approximate height of P) it is easy to draw OP'' parallel to the tangent at P to the refraction curve, SPO. This gives P'', necessarily below P. Hence P lies somewhere between P' and P''.

A more exact determination could be had by drawing the curve of refraction, OPS, corresponding to the angle r and noting its intersection with the normal at D.

If the observer happens to be at P, the point O will appear elevated to O'. Clearly, however, from a knowledge of the angle $PO'N$, and the approximate air densities at P and O, one may draw $P[illegible]$ parallel to $P'O$, and thus locate O somewhere between the two determined points O' and O''. Here, too, it would be more accurate to use the refraction curve, SPO.

Even initially horizontal rays normally curve down towards the surface of the earth, so that objects at the observer's own level, as well as those above and below it, appear elevated. To understand this phenomenon, consider a wave front normal to the surface of the earth and, consequently, moving horizontally. If, now, the density of the air at the place in question decreases with increase of elevation, as it nearly always does, the upper portion of the wave front will travel faster than the lower, and the path will be bent down towards the earth along a curve whose radius depends upon the rate of this density decrease. For example, let the corrected height of the barometer be 760 mm., the temperature 17° C., and the rate of temperature decrease with elevation 5° C. per kilometer; conditions that not infrequently obtain at sea level. On substituting these values in the density-elevation equation, it appears that the density gradient would be such that if continuous the limit of the atmosphere would be reached at an elevation of about 10 kilometres. Hence, under these circumstances, the velocity of light at an elevation of 10 kilometres would be to its velocity at the surface in the ratio of 1,000,276 to 1,000,000, approximately, since the refractive index of the lower air would be 1,000,276, about. The radius of curvature, r, therefore, is closely given in kilometres by the equation,

$$\frac{r}{r + 10} = \frac{1{,}000{,}000}{1{,}000{,}276}.$$

Hence, $r = 36{,}232$ kilometres, or approximately 5.7 times the radius of the earth.

It is conceivable, therefore, that the size of a planet and the vertical density gradient of its atmosphere might be such that one's horizon on it would include the entire surface—that he could look all the way round and, as some one has said, see his own back.

The distance to the horizon, corresponding to a given altitude, obviously depends upon the rate of vertical density decrease in such manner that when the latter is known the approximate value of the former can easily be computed. Thus, let the density decrease be such that the radius of curvature of a ray tangent to the surface shall be $5.7\ R$, $R = 6366$ kilometres, being the radius of the earth; let α be the angle betwen the radii from the centre of the earth to the observer and a point on his unobstructed horizon respectively; let h be the observer's height in metres above the level of his horizon; and let r be the distance, in kilometres, measured over the surface from the horizon to a point on the same level below the observer; then, by trigonometry, to a close approximation;

$$h = 6366000\ (\sec\alpha - 1) - 36286200\ (\sec\frac{\alpha}{5.7} - 1)\sec\alpha$$

and

$$r = 6366\alpha,\ \alpha \text{ in radians}$$

A few values of the distance to the horizon from different elevations, computed by the above formula, are given in the following table:

Distance to Horizon.

Distance in Kilometres.	1	2	5	10	20	50	100
Elevation in metres.	.061	.263	1.613	6.856	25.901	161.918	647.604

Looming.—Since the extension of the actual beyond the geometrical horizon depends, as just explained, upon the density decrease of the atmosphere with increase of elevation, it follows that any change in the latter must produce a corresponding variation of the former. An increase, for instance, in the normal rate of decrease, such as often happens over water in middle to high latitudes, produces the phenomenon of looming, or the coming into sight of objects normally below the horizon, a classical in-

stance of which was described by Latham.[169] Similar changes in the rate of density decrease with increase of elevation also are common in valleys, but here looming, in the above sense, is rendered impossible by the surrounding hills or mountains.

Towering.—The condition of the atmosphere that produces looming, in the sense here used, or would produce it if the region were level, often gives rise to two other phenomena, namely, unwonted towering, also usually called looming, and the consequent apparent approach of surrounding objects.

The more rapid the downward curvature of the ray paths at the observer the more elevated, clearly, will objects appear to be, and such curvature may, indeed, be very considerable. Thus, a temperature inversion near the surface of the earth of 1° C. per metre change of elevation bends down a ray along an arc whose radius is about 0.16 that of the earth, while an inversion of 10° C. per metre—a possible condition through a shallow stratum—gives a radius of only about 0.016 that of the earth, or, say, 100 kilometres. If now, as occasionally happens, the inversion layer is so located that rays to the observer from the top of an object are more curved than those from the bottom, it will appear not only elevated but also vertically magnified—it will tower and seem to draw nearer.

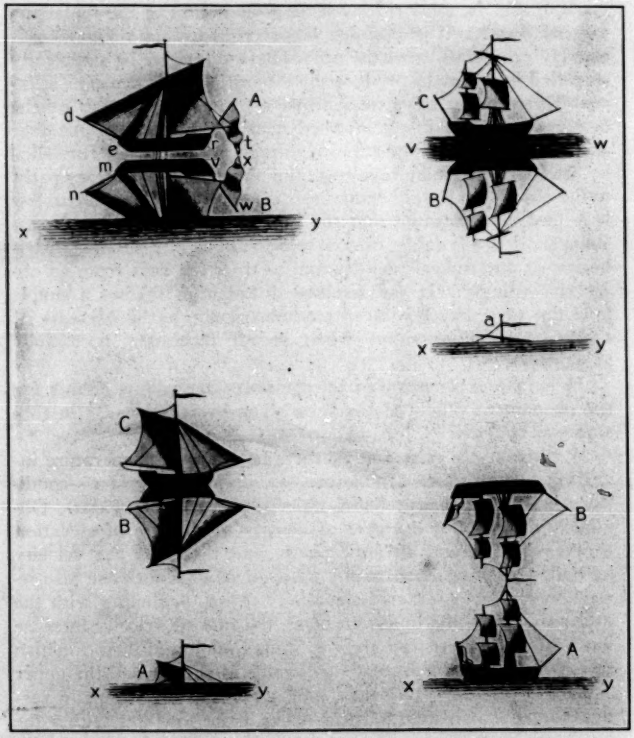
Sinking.—Instead of increasing the curvature of rays the temperature distribution may be such as, on the contrary, to decrease it and thereby cause objects normally on the horizon to sink quite beyond it. Such phenomena, exactly the reverse of looming, are also most frequently observed at sea.

Stooping.—Occasionally rays from the base of an object may be curved down much more rapidly than those from the top, with the obvious result of apparent vertical contraction, and the production of effects quite as odd and grotesque as those due to towering. Indeed, since the refraction of the atmosphere increases, in general, with the zenith distance, it is obvious that the bases of objects nearly always apparently are elevated more than their tops, and themselves, therefore vertically shortened. The normal effect, however, is small and seldom noticed except, perhaps, in connection with the slightly flattened shape of the sun and moon when on the horizon.

[169] *Phil. Trans.*, v. **88**, 1798; abridged, v. **18**, p. 337. See also Everett, "Nature," v. 11, p. 49, 1874.

Superior Mirage.—It occasionally happens that one or more images of a distant object, a ship for instance, are seen directly

FIG. 130.



Examples of superior mirage (Vince).

above it, as shown in Fig. 130, copied from Vince's [170] well-known description of exceptionally fine displays of this phenomenon.

[170] *Phil. Trans.*, **89** (8, abridged, p. 436), 1799.

The mirage nearest the object always is inverted and therefore appears as though reflected from an overhead plane mirror—hence the name "superior mirage"—and, indeed, many seem to assume that this image really is due to a certain kind of reflection, that is, total reflection, such as occurs at the undersurface of water. It is obvious, however, that this assumption is entirely erroneous, since the atmosphere can never be sufficiently stratified in nature to produce the discontinuity in density (adjacent layers are always interdiffusible) this explanation of the origin of the proximate inverted image presupposes. Another apparently simple explanation of mirage phenomena is furnished by drawing imaginary rays from the object along arbitrary paths to the observer. But, in reality, this is no explanation at all, unless it is first demonstrated that the rays must follow the paths assumed. It is allowable, of course, to assume any *possible* distribution of atmospheric density and to trace the rays from an object accordingly. If the assumed distribution follows a simple law, the rays may be traced mathematically, as by Mascart,[171] though such discussions, when at all thorough, necessarily are long.

A simple explanation of mirage that admirably accounts for the phenomena observed has been given by Hastings,[172] in substance as follows:

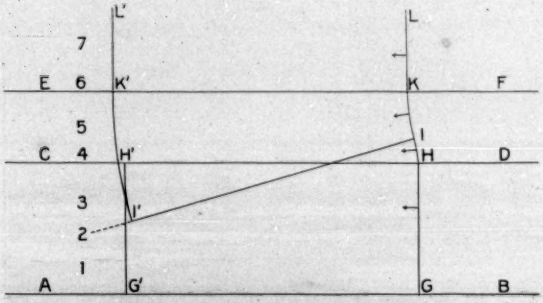
Let the air be calm and let there be a strong temperature inversion some distance, 10 metres, say, above the surface—conditions that occasionally obtain, especially over quiet water. Obviously the ratio of decrease of density to increase of elevation is irregular in such an atmosphere, and therefore the velocity of light travelling horizontally through it must increase also irregularly with increase of elevation. Thus, beginning with the undersurface of the inversion layer, the rate of velocity increase with elevation must first grow to a maximum and then diminish to something like its normal small value at and beyond the upper surface of this layer. Hence, that portion of an originally vertical, or approximately vertical, wave front that lies within the inversion layer must soon become doubly deflected, substantially as indicated in Fig. 131.

[171] "Traité d'Optique," v. 3, pp. 305–308.

[172] "Light," Chapter 7, New York, Chas. Scribner's Sons, 1902.

Let *AB* (Fig. 131) be the surface of water, say, *CD* the under and *EF* the upper surface of a strong inversion layer, and let *GHIKL* be the distorted wave front of light, travelling in the direction indicated, from a distant source. The future approximate positions of the wave front, of which *G'H'I'K'L'* is one, are readily located from the fact that its progress is always normal to itself, and the appearance of the distant object from which these wave fronts are proceeding easily determined. At 1, for instance, the object seems upright and at its proper level, no images are seen and the whole appearance is normal; at 2 confused elevated images appear, in addition to the object itself; at 3 the

FIG. 131.



Wave fronts giving a superior mirage (Hastings).

object and 2 distinct images are seen, the lower produced by the segment *HI* of the wave front inverted, the other erect; at 4, the under surface of the inversion layer, the inverted image blends with the object and disappears; at 5 only the erect image can be seen, and, indeed, may be seen even when the object itself normally would be below the horizon; at 6, the upper surface of the inversion layer, the vertical image merges with the object and disappears; while everywhere beyond the upper surface only the object itself is visible, as at 1, with no evidence whatever of abnormal refraction and mirage.

An additional inversion layer obviously might produce other images, while more or less confused layers might produce multiple and distorted images, such as shown in Fig. 132, copied from

Scoresby's account [173] of a certain telescopic view of the east coast of Greenland.

Inferior Mirage.—It is a very common thing in flat desert regions, especially during the warmer hours of the day, to see below distant objects and somewhat separated from them their apparently mirrored images—the inferior image. The phenomenon closely simulates, even to the quivering of the images, the reflection by a quiet body of water of objects on the distant shore, the "water" being the image of the distant low sky, and therefore frequently leads to the false assumption that a lake or bay

FIG. 132.

Telescopic appearance of the coast of Greenland, at the distance of 35 miles, when under the influence of an extraordinary refraction. July 18, 1820. Lat. 71° 20′, Long. 17° 30′ W.

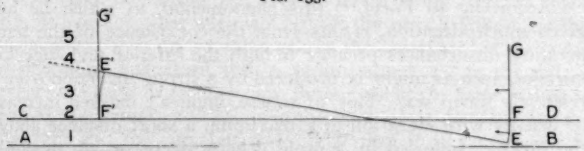
is close by. This type of mirage is very common on the west coast of Great Salt Lake. Indeed, on approaching this lake from the west one can often see the railway over which he has just passed apparently disappearing beneath a shimmering surface. It is also common over smooth-paved streets provided one's eyes are just above the street level. An under-grade crossing in a level town, for instance, offers an excellent opportunity almost any warm day of seeing well-defined small images that are apt to arouse one's surprise at the careless way his fellow-citizens wade through pools of water!

[173] *Trans. Roy. Soc. Edinburgh*, v. 9, p. 299, 1823.

Since the inferior mirage occurs only over approximately level places and there only when they are so strongly heated that for a short distance the density of the atmosphere increases with elevation it follows that its explanation is essentially the same as that of the superior mirage. Of course, the surface air is in unstable equilibrium and rising in innumerable filaments, but its rarefied state is maintained so long as there is an abundant supply of insolation. A wave front, therefore, from an object slightly above the general level soon becomes distorted through the greater speed of its lower portion, as schematically indicated in Fig. 133.

Let *AB* of this figure be the surface of the earth and *CD* the upper level of the superheated stratum. Let *EFG* be the position of a wave front travelling as indicated, the lower portion curved forward as a result of its greater speed in the rarefied

FIG. 133.



Wave fronts giving an inferior mirage (Hastings).

layer. One of the consequent later positions of the wave front is shown at *E'F'G'*, from which it follows that at 1 neither the object in question nor any image of it can be seen; at 2 the object and its inverted image are glimpsed, superimposed; at 3 both the object and its inverted image, well below it, are plainly visible; at 4 the image is just disappearing; while at 5 there is no evidence of a mirage, unless of objects more distant than the one under consideration.

Great uncertainty may exist, therefore, in regard to the exact positions of objects seen in (or perhaps hidden by) a mirage. Thus, in his official report of the battle of April 11, 1917, between the English and the Turks in Mesopotamia, General Maude, the British commander, says: "The fighting had to be temporarily suspended owing to a mirage."

Lateral Mirage.—Vertical sheets of abnormally dense or abnormally rare atmosphere obviously would produce lateral mirages in every way like those due to similar horizontal layers,

and indeed such mirages are occasionally seen along walls and cliffs whose temperatures differ widely from that of the air a few metres from them.

Fata Morgana.—Morgana (Breton equivalent of sea woman), according to Celtic legend and Arthurian romance, was a fairy, half-sister of King Arthur, who exhibited her powers by the mirage. Italian poets represent her as dwelling in a crystal palace beneath the waves. Hence, presumably, the name Fata Morgana (Italian for Morgan le Fay, or Morgan the fairy) was given, centuries ago, to those complicated mirages that occasionally appear over the strait of Messina moulding the bluffs and houses of the opposite shore into wondrous castles that alike tower into the sky and sink beneath the surface; nor is it strange that this poetical name should have become generic, as it has, for all such multiple mirages wherever they occur.

According to Forel,[174] this phenomenon, to which he has given much attention, results from the coexistence of the temperature disturbances peculiar to both the inferior and superior mirages, such as might be produced by a strong inversion over a relatively warm sea. This, of course, implies a marked increase of density with elevation to a maximum a short distance above the surface, followed by a rapid density decrease—an unstable condition and therefore liable to quick and multiform changes. Obviously, too, such a cold intermediate layer in addition to producing a double mirage acts also as a sort of cylindrical lens that vertically magnifies distant objects seen through it.

No wonder, then, that under such circumstances the most commonplace cliffs and cottages are converted, through their multiple, distorted, and magnified images, into magic castles, or the marvellous crystal palaces of Morgan le Fay!

CHAPTER III.

REFRACTION PHENOMENA: REFRACTION BY WATER DROPS—RAINBOW.

Principal Bows.—It may seem entirely superfluous to describe so common a phenomenon as the rainbow, or to offer more than the simple explanation of it that may be found in innumerable text-books. But rainbows differ among themselves as one tree

[174] *Archives des Sciences Phys. et Nat.*, v. 32, p. 471, 1911.

from another, and besides some of their most interesting features usually are not even mentioned—and naturally so, for the "explanations" generally given of the rainbow may well be said to explain beautifully that which does not occur and to leave unexplained that which does.

The ordinary rainbow, seen on a sheet of water drops—rain or spray—is a group of circular or nearly circular arcs of colors whose common centre is on the line connecting the observer's eye with the exciting light (sun, moon, electric arc, etc.) or rather, except rarely, on that line extended in the direction of the observer's shadow. A very great number of rainbows are theoretically possible, as will be explained later, and doubtless all that are possible actually occur, though only three (not counting supernumeraries) certainly have been seen on sheets of rain. The most brilliant bow, known as the *primary*, with red outer border of about 42° radius and blue to violet inner border, appears opposite the sun (or other adequate light); the next brightest, or the *secondary* bow, is on the same side of the observer, but the order of its colors is reversed and its radius, about 50° to the red, is larger; the third or *tertiary* bow, having about the same radius as that of the primary and colors in the same order, lies between the observer and the sun, but is so faint that it is rarely seen in nature. Obviously the common centre of the primary and secondary bows is angularly as far below the observer as the source (sun generally) is above, so that usually less than a semicircle of these spectral arcs is visible, and never more, except from an eminence.

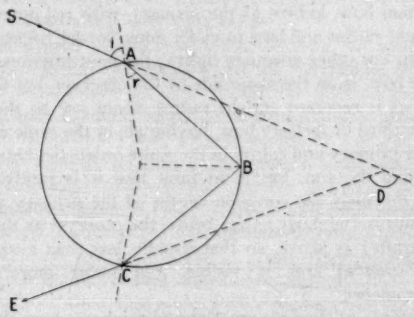
The records of close observations of rainbows soon show that not even the colors are always the same; neither is the band of any color of constant angular width; nor the total breadth of the several colors at all uniform; similarly the purity and brightness of the different colors are subject to large variations. The greatest contrast, perhaps, is between the sharply-defined brilliant rainbow of the retreating thunderstorm and that illy-defined faintly tinged bow that sometimes appears in a mist—the "white bow" or "fog bow."

All these differences depend, as will be explained later, essentially upon the size of the drops, and therefore inequalities often exist between even the several portions, especially top and bottoms, of the same bow, or develop as the rain progresses. Ad-

ditional complications occasionally result from the reflection of bows and from bows produced by reflected images of the sun, but though unusual and thus likely to excite wonder and comment such phenomena are easily explained.

Supernumerary Bows.—Rather narrow bands of color, essentially red, or red and green, often appear parallel to both the primary and the secondary bows, along the inner side of the first and outer of the second. These also differ greatly in purity and color, number visible, width, etc., not only between individual bows but also between the several parts of the same bow. No such colored arcs, however, occur between the principal bows; indeed, on the contrary, the general illumination here is perceptibly at a minimum.

FIG. 134.



Change in light direction by raindrop.

Deviation in Direction of Emerging from Entering Ray.—Since a raindrop is spherical, its action on an enveloping wave front may be obtained by determining first the effects in the plane of a great circle containing an entering ray, and then revolving this plane about that line in it that bisects the angle between the incident and emerged paths of any given ray in the same plane. Let, then, *ABC* (Fig. 134) be the plane of a great circle of an enlarged raindrop and let *S A B C E* be the path of a ray in this plane, entering the drop at *A* and emerging at *C*. The changes in direction at *A* and *C* are each $i - r$, in which i is the angle of incidence and r the angle of refraction, and the change at *B*, as

also at every other place of an internal reflection, when there are more than one, is $\pi - 2r$. Hence, the total deviation, D, is given by the equation,

$$D = 2(i - r) + n(\pi - 2r) \quad \ldots\ldots\ldots\ldots (1)$$

in which n is the number of internal reflections.

Minimum Deviation.—The above general expression for the deviation shows that it varies with the angle of incidence. There is also a minimum deviation, corresponding to a particular angle of incidence, as may be shown in the usual way. Thus by equation (1),

$$dD = 2di - 2(n + 1)\,dr$$

which, on putting $dD = O$, the condition of stationary (maximum or minimum) deviation, gives,

$$di = (n + 1)\,dr \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

But $\sin i = \mu \sin r$, in which μ is the refractive index of water with reference to air, and, therefore,

$$\cos i\, di = \mu \cos r\, dr.$$

Hence, by (2)

$$\mu \cos r = (n + 1) \cos i$$

and

$$\cos i = \sqrt{\frac{\mu^2 - 1}{n^2 + 2n}}.$$

This value of the angle of incidence corresponds, as stated, to a stationary deviation, but whether of maximum or minimum value may be determined by noting the sign of the second differential, which gives:

$$\frac{d^2D}{di^2} = -2(n + 1)\frac{d^2r}{di^2}$$

But

$$\frac{dr}{di} = \frac{\cos i}{\mu \cos r},$$

and

$$\frac{d^2r}{di^2} = \frac{(1 - \mu^2)\sin i}{\mu^3 \cos^3 r},$$

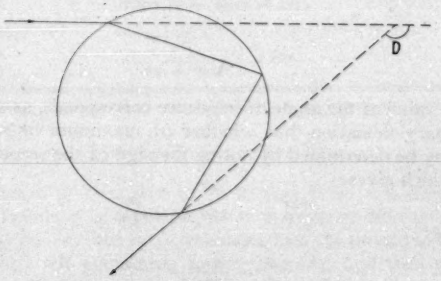
Hence, as μ is greater than unity, this latter value is negative and therefore the second differential of D with respect to i is positive, and the corresponding value of D is a minimum.

The following table gives these values for the primary, secondary, tertiary, quaternary, and quinary rainbows corresponding to 1, 2, 3, 4, and 5 internal reflections, respectively, as shown in Figs. 135, 136, 137, 138, and 139.

		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
Violet, H	i	58° 48′	71° 30′	76° 36′	79° 27′	81° 17′
$\lambda=3968.5$	r	39° 33′	44° 54′	46° 23′	47° 2′	47° 22′
$\mu=1.3435$	D	$\pi-40°\ 36'$	$2\pi-126°\ 24'$	$2\pi-37°\ 52'$	$3\pi-131°\ 26'$	$3\pi-45°\ 50'$
Yellow	i	59° 23′	71° 50′	76° 50′	79° 38′	81° 26′
$\lambda=5800.0$, about	r	40° 12′	45° 27′	46° 55′	47° 32′	47° 52′
$\mu=4/3$	D	$\pi-42°\ 2'$	$2\pi-129°\ 2'$	$2\pi-41°\ 40'$	$3\pi-136°\ 4'$	$3\pi-51°\ 32'$
Red, $H\alpha$	i	59° 31′	71° 54′	76° 53′	79° 40′	81° 28′
$\lambda=6562.9$	r	40° 21′	45° 34′	47° 2′	47° 39′	47° 59′
$\mu=1.3311$	D	$\pi-42°\ 22'$	$2\pi-129°\ 36'$	$2\pi-42°\ 30'$	$3\pi-137°\ 10'$	$3\pi-52°\ 52'$

Entering and Emerging Rays.—Since a raindrop is spherical, it is obvious that its effect on incident radiation from the sun, or other spherical or point source, is symmetrical about an axis

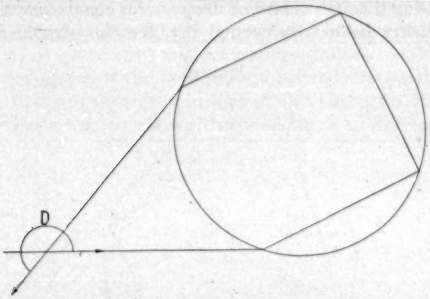
FIG. 135.



Change of light direction in primary rainbow.

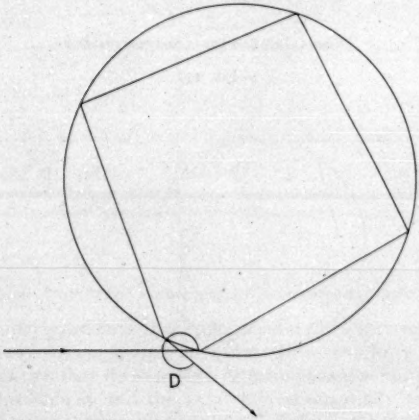
through the centre of the drop and the luminous object. Hence, in the study of the rainbow, it is sufficient to use only a single plane containing this axis, tracing the rays incident over one quadrant of the intersection circle and noting the resulting phenomena. It is also obvious that, neglecting sky light, solar rays

FIG. 136.



Change of light direction in secondary rainbow.

FIG. 137.

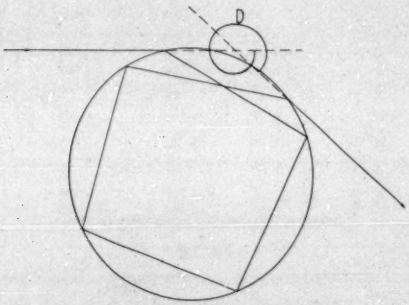


Change of light direction in tertiary rainbow.

are parallel to within the angular diameter of the sun, 0.5° about, and that as a first approximation they may be regarded as strictly parallel. Let, then, the plane of Fig. 140 pass through the centres of a raindrop and the sun, and let *AB* be the wave front

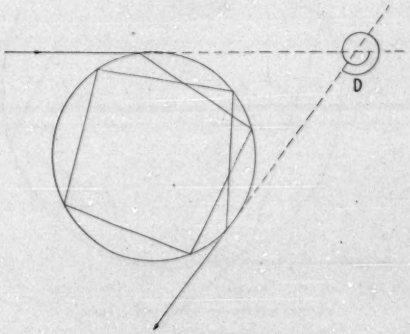
of parallel rays incident, as shown, above the normal or axial ray (ray passing through centre of drop). An equal amount of light clearly enters below the normal ray, but for simplicity this is

FIG. 138.



Change of light direction in quaternary rainbow.

FIG. 139.

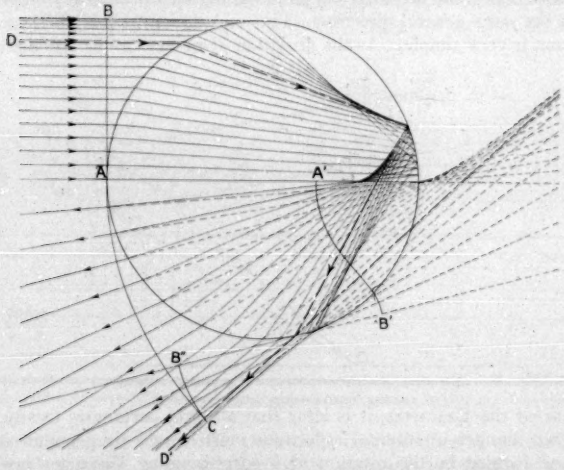


Change of light direction in quinary rainbow.

omitted. Similarly that portion reflected from the outer surface is ignored, as is also all that is internally reflected more than once. This reduces the problem of the rainbow to its simplest terms, but loses none of its generality, since additional internal

reflections merely change angular dimensions and brightness. The heavy line shows the course of the Descartes ray, or ray of minimum deviation for light of air-water refractive index, 4/3. The courses of other rays are, very approximately, as indicated. Since the deviations of the rays incident between the axial and the Descartes rays are greater than that of the Descartes, it follows that their exits are, as shown, between those of the same two

FIG. 140.



Course of light through a raindrop and the corresponding wave fronts.

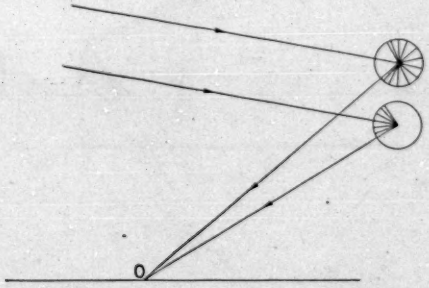
rays. Similarly, all rays that enter beyond the Descartes ray are likewise more deviated, and, therefore, while they leave the drop beyond this ray, they do so in such direction as sooner or later also to come between it and the axial ray, substantially as shown. Clearly, then, the once reflected light is diffuse and feeble except near the path of minimum deviation, and confined, as indicated, to the region between this path and the axial ray.

Formation of the Bow.—From the course, just given, of light through raindrops, it is clear that maximum brightness will be produced by all illuminated drops along the elements of a right

circular cone whose vertex is at the eye, whose axis passes through the sun, and whose angular opening, corresponding to a given number of internal reflections, is determined by the wave-length. Hence, the rainbow exhibits a number of concentric circular arcs of different colors whose centres are angularly as far below the observer as the sun is above him.

Minimum Brightness Between Primary and Secondary Bows.—Careful observers often note the fact that the region between the primary and secondary bows is slightly darker than any other in the same general direction. The explanation of this phenomenon is very simple. As the deviation of no ray can be less than

FIG. 141.



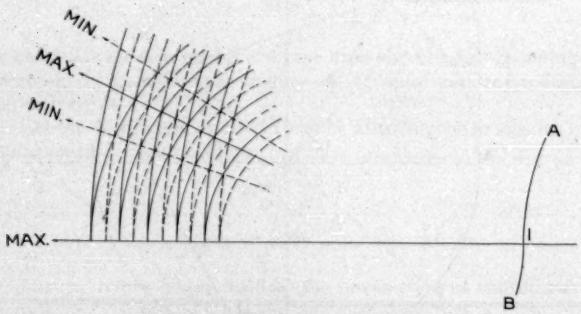
Minimum light between primary and secondary rainbows.

that of the Descartes, it is clear that all light pertaining to any given number of internal reflections must leave a drop within a cone formed by the rotation of a corresponding Descartes ray about the axial ray, keeping the angle between them constant as shown in Fig. 141 for light of 1, and 2 internal reflections, respectively. Hence, once reflected light reaches an observer from drops along and within his primary bow, but none from those without, while twice reflected light reaches him from along and without the secondary bow, but none from within it. The region between the two bows, therefore, and it alone, is devoid of both the once and the twice reflected rays and in consequence is comparatively dark.

Origin of Supernumerary Bows.—Since wave fronts are normal to the corresponding rays, it is clear that the incident

front, AB (Fig. 140), will, at the moment of complete emergence, appear as ACB''—exactly as though it had come from the virtual front, $A'B'$, the locus of the terminus of a line of constant length, AA', as it travels normally over the emerging wave front, ACB''. Further, since the rays here lie on both sides of the one of minimum deviation, it is obvious that this ray divides $A'B'$ into two portions curved in opposite directions. That portion of the front that is convex forward will, of course, remain convex, but with increasing radii of curvature, while the part that is concave forward will later become convex, and although neither portion is

FIG. 142.



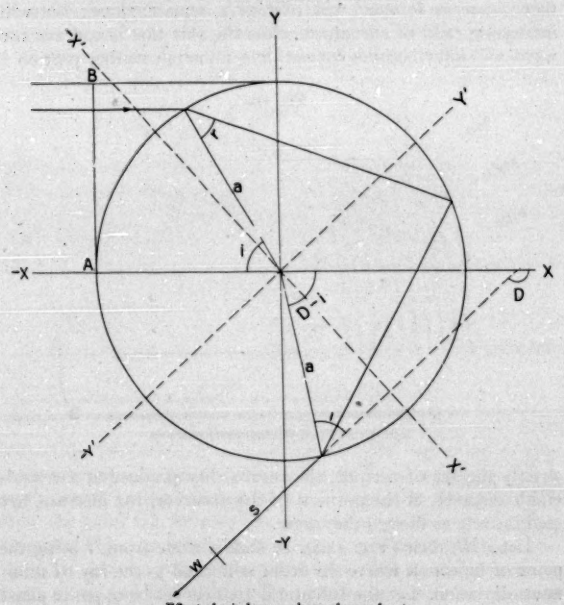
Interference giving supernumerary rainbows.

strictly the arc of a circle, the results they produce at a considerable distance, at the position of the observer, for instance, are qualitatively as though they were.

Let AIB, then (Fig. 142), be such a wave front, I being the point of inflection where the front is normal to the ray of minimum deviation. Let the full and dotted curves be opposite phase positions of the resulting cusped wave front. By inspection, it is obvious that soon after leaving the drop all the light must lie on one side of the ray of minimum deviation, thus making the observed angular radius of an arc of any given color slightly less than that of the Descartes ray. It is also obvious that with increasing angular distance from the Descartes ray the two branches of the cusped front are alternately, and with increasing

frequency, in opposite and like phases, thus producing alternate arcs of minimum and maximum brightness, all of smaller angular radius than that of the first maximum, the chief bow, and progressively lying closer and closer together as the radius diminishes, or increases, in the case, respectively, of the primary and the secondary bow.

FIG. 143.



Effect of raindrop on shape of wave front.

These additional maxima, of which several frequently are visible, constitute the familiar supernumerary bows.

Equation of Portion of Outgoing Wave Front Near Ray of Minimum Deviation.—In deriving this equation, originally due to Airy,[175] the more direct and elementary method of Wirt-

[175] *Cambridge Phil. Trans.*, vi, 380.

inger [176] will be followed with only such modifications as appear to make for clearness.

Let AB (Fig. 143) be an incident wave front tangent to a raindrop of radius a; let w be a small section of the emitted front near the minimum deviation ray—more distant portions of the front need not be considered as they are formed by rays too divergent to produce anything more than a slight general illumination; let v_1 be the velocity in air of the light under examination and $v_1\mu$ its velocity in water. Clearly, then, from the constancy of the time interval between corresponding points on the wave fronts,

$$\frac{a\ (1 - \cos i)}{v_1} + \frac{4\mu\ a\ \cos r}{v_1} + \frac{s}{v_1} = T, \text{ a constant.}$$

in which s is the distance between the drop and the corresponding point on w; i, and r, the angles of incidence and refraction, respectively.

Let the completely emitted front be also tangent to the drop, as shown in Fig. 140. Then

$$T = \frac{4\mu\ a}{v_1}$$

and

$$s = a\left\{4\mu\ (1 - \cos r) - (1 - \cos i)\right\}$$

Let the centre of the drop be the intersection of coördinates as indicated. Then, if D is the ang'e of deflection, a point on w is given by the values,

$$x = a \cos (D - i) + s \cos D,$$

and

$$y = -a \sin (D - i) - s \sin D.$$

By turning the coördinates clockwise through the angle $D_1 - \pi/2$, in which D_1 is the change in direction of the ray of minimum deviation, the projection angles are correspondingly reduced and the new y axis brought parallel to the emerged Descartes ray. Hence, in terms of the new coördinates, writing d for $D - D_1$,

$$x' = -a \sin (d - i) - s \sin d$$
$$-y' = a \cos (d - i) + s \cos d.$$

[176] Berichte des naturw. medicin. Vereines an der Universität Innsbruck, xxi Jahrg., 1896–97. See also Pernter-Exner, Meteorologische Optik.

But as only rays very near that of minimum deviation need be considered, d is so small that to a sufficiently close approximation,

$$\cos d = 1, \text{ and } \sin d = d.$$

Hence,

$$x' = -ad \cos i + a \sin i - sd,$$

and

$$-y' = a \cos i + a d \sin i + s.$$

From Fig. 140 it is obvious that the small section, w (Fig. 143), of the emerged wave front is very nearly parallel to the x' axis, and that the x' coördinate, therefore, is extremely sensitive to changes in y', while the y' coördinate is relatively but little affected by the changes in x'. Hence, as d is very small, points on w are sufficiently closely given by the expressions,

$$x' = a \sin i$$

and

$$-y' = a \cos i + ad \sin i + a \left\{4\mu (1 - \cos r) - (1 - \cos i)\right\}.$$

Let I and R be the angles of incidence and refraction, respectively, corresponding to the minimum deviation, D_1, and let

$$i = I + \alpha, \text{ and } r = R + \beta$$

in which α and β are quite small, since only rays near that of minimum deviation are considered. Further, to make the problem entirely general, let n be the number of internal reflections. Then,

$$x' = a \sin (I + \alpha) = a \sin I + a\alpha \cos I$$

$$-y' = a \cos i + ad \sin i + a \left\{2 \mu (n + 1) (1 - \cos r) - (1 - \cos i)\right\}$$

$$= 2a \left\{\cos i + \frac{d}{2} \sin i - \mu (n + 1) \cos r + \mu (n + 1) - \tfrac{1}{2}\right\}.$$

Treating d and β as functions of α and developing we get, neglecting powers of α higher than the third,

$$\cos i = \cos (I + \alpha) = \cos I - \alpha \sin I - \frac{\alpha^2}{2} \cos I + \frac{\alpha^3}{6} \sin I + \cdots\cdots$$

$$\frac{d}{2} = \tfrac{1}{2} (D - D_1) = (I + \alpha) - (R + \beta) + \frac{n}{2} \left\{\pi - 2 (R + \beta)\right\} - \left\{I - R + \frac{n}{2} (\pi - 2R)\right\}$$

$$= \alpha - (n + 1) \beta = \alpha - (n + 1) \left(\frac{d\beta}{d\alpha}\right)_0 \alpha - (n + 1) \left(\frac{d^2\beta}{d\alpha^2}\right)_0 \frac{\alpha^2}{2} - (n + 1) \left(\frac{d^3\beta}{d\alpha^3}\right)_0 \frac{\alpha^3}{6}$$

Since

$$\frac{d}{2} = \alpha - (n+1)\beta,\ d\frac{d}{2} = d\alpha - (n+1)\,d\beta, \text{ and } \left(\frac{d\beta}{d\alpha}\right)_0 = \frac{1}{n+1},$$

and, since

$$i = I + \alpha, \text{ or } \sin i = \sin I + \alpha \cos I,$$

therefore,

$$\frac{d}{2}\sin i = -(n+1)\left(\frac{d^2\beta}{d\alpha^2}\right)_0 \frac{\alpha^2}{2}\sin I - (n+1)\left(\frac{d^3\beta}{d\alpha^3}\right)_0 \frac{\alpha^3}{6}\sin I - (n+1)\left(\frac{d^2\beta}{d\alpha^2}\right)_0 \frac{\alpha^3}{2}\cos I \text{ ----}$$

Finally,

$$\cos r = \cos\left\{R + f(\alpha)\right\} = \cos R + \alpha\frac{d\cos r}{d\alpha} + \frac{\alpha^2}{2}\frac{d^2\cos r}{d\alpha^2} + \frac{\alpha^3}{6}\frac{d^3\cos r}{d\alpha^3} + \text{-----}$$

But

$$\frac{d\cos r}{d\alpha} = -\sin r\frac{d\beta}{d\alpha} = -\frac{\sin i}{\mu}\frac{d\beta}{d\alpha},$$

$$\frac{d^2\cos r}{d\alpha^2} = -\frac{\cos i}{\mu}\frac{d\beta}{d\alpha} - \frac{\sin i}{\mu}\frac{d^2\beta}{d\alpha^2}$$

$$\frac{d^3\cos r}{d\alpha^3} = \frac{\sin i}{\mu}\frac{d\beta}{d\alpha} - \frac{\cos i}{\mu}\frac{d^2\beta}{d\alpha^2} - \frac{\cos i}{\mu}\frac{d^2\beta}{d\alpha^2} - \frac{\sin i}{\mu}\frac{d^3\beta}{d\alpha^3}$$

At the limits

$$\left(\frac{d\beta}{d\alpha}\right)_0,\ \left(\frac{d^2\beta}{d\alpha^2}\right)_0,\ \left(\frac{d^3\beta}{d\alpha^3}\right)_0, \text{ where } i = I,$$

$$-\mu(n+1)\cos r = -\mu(n+1)\cos R - (n+1)\left(\frac{d\beta}{d\alpha}\right)_0\left[-\alpha\sin I - \frac{\alpha^2}{2}\cos I + \frac{\alpha^3}{6}\sin I\right] - (n+1)\left(\frac{d^2\beta}{d\alpha^2}\right)_0\left[-\frac{\alpha^2}{2}\sin I - \frac{\alpha^3}{3}\cos I\right] + (n+1)\left(\frac{d^3\beta}{d\alpha^3}\right)_0\left[\frac{\alpha^3}{6}\sin I\right]$$

Hence, by addition of the above terms, remembering that,

$$\mu\cos R = (n+1)\cos I$$

and that

$$\left(\frac{d\beta}{d\alpha}\right)_0 = \frac{1}{n+1}$$

$$-y' = 2a\left\{1 - (n+1)^2\right\}\cos I - a(n+1)\left(\frac{d^2\beta}{d\alpha^2}\right)_0\frac{\alpha^2}{3}\cos I + a\left\{2\mu(n+1) - 1\right\}$$

Since

$$\sin i = \mu \sin r$$

$$\cos i = \mu \cos r \frac{d\beta}{d\alpha}$$

and

$$-\sin i = -\mu \sin r \left(\frac{d\beta}{d\alpha}\right)^2 + \mu \cos r \left(\frac{d^2\beta}{d\alpha^2}\right)$$

or

$$-\sin I = -\mu \sin R \left(\frac{d\beta}{d\alpha}\right)^2_0 + \mu \cos R \left(\frac{d^2\beta}{d\alpha^2}\right)_0$$

$$= -\frac{\mu \sin R}{(n+1)^2} + \mu \cos R \left(\frac{d^2\beta}{d\alpha^2}\right)_0$$

But

$$\mu \sin R = \sin I, \text{ and } \mu \cos R = (n+1) \cos I.$$

Therefore,

$$\left(\frac{d^2\beta}{d\alpha^2}\right)_0 = -\frac{\sin I}{\cos I} \frac{n^2+2n}{(n+1)^2}$$

and

$$-y' = 2a\left[-(n^2+2n)\cos I + \mu(n+1) - \tfrac{1}{2}\right] + a\alpha^2 \frac{n^2+2n}{3(n+1)^2}$$

$$\sin I, \quad x' = \left[a \sin I\right] + a\alpha \cos I.$$

The first terms in the equations for x' and y' are the coördinates of the point of inflection on the curve w. Taking this point as the origin and calling the coördinates x_1 and y_1 we have for points on w,

$$x_1 = a\alpha \cos I$$

$$y_1 = a\alpha^2 \frac{n^2+2n}{3(n+1)^2} \sin I.$$

But

$$\alpha^2 = \frac{x_1^2}{a^2 \cos^2 I},$$

hence,

$$y_1 = \frac{1}{3a^2} \frac{n^2+2n}{(n+1)^2 \cos^2 I} x_1^2 \sin I.$$

As previously shown,

$$\cos I = \sqrt{\frac{\mu^2-1}{n^2+2n}},$$

hence,

$$y_1 = \frac{1}{3a^2} \frac{(n^2+2n)^2}{(n+1)^2(\mu^2-1)} \sqrt{\frac{(n+1)^2-\mu^2}{\mu^2-1}}\, x_1^3.$$

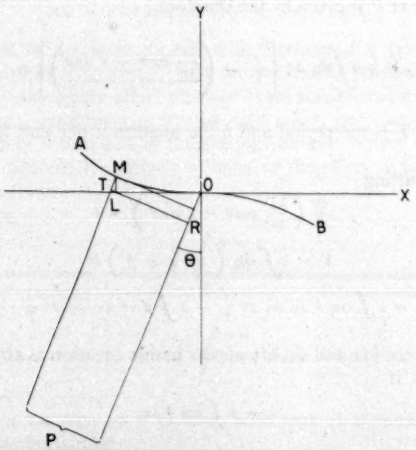
Putting

$$\frac{(n^2 + 2n)^2}{(n + 1)^2 (\mu^2 - 1)} \sqrt{\frac{(n + 1)^2 - \mu^2}{\mu^2 - 1}} = h,$$

$$y_1 = \frac{h}{3a^2} x_1^3.$$

This equation, then, represents a curve very nearly coincident with that portion of the wave front to which the rainbow phenomena are due, and since the effects computed from it substantially agree with those observed, as will be seen presently, it is clear that the approximation thus obtained is sufficient for most, if not all, practical uses; indeed, the assumption that raindrops are perfectly spherical involves, perhaps, a greater error.

FIG. 144.



Variation of intensity with angular distance from ray of minimum deviation.

Intensity and Its Variation with Angular Distance from the Ray of Minimum Deviation.—This, too, was first determined by Airy.[177] The following discussion, however, is essentially that of Mascart [178] and Pernter-Exner.[179]

[177] *l. c.*

[178] *Traité d'Optique*, i.

[179] *Meteorologische Optik.*

Let O (Fig. 144) be the point of inflection of an emitted wave front near a drop; let P be a distant point in the direction θ from the ray of minimum deviation. Then the difference in phase, ΔF, between the light vibrations at P, due respectively to an element, ds, of the front at O and an equal element at M is given by the equation,

$$\Delta F = 2\pi \frac{OR - MT}{\lambda} = 2\pi \frac{x \sin\theta - y \cos\theta}{\lambda}$$

in which λ is the wave-length.

Hence, substituting for y its value, $\frac{hx^3}{3a^2}$, and dx for ds, which is allowable over the effective portion of the wave front, the vibration at P is given by the equation,

$$V = k\int \sin 2\pi \left[\frac{t}{T} - \left(\frac{\frac{hx^3}{3a^2}\cos\theta - x\sin\theta}{\lambda}\right)\right] dx$$

in which T is the period and k the amplitude per unit length of front.

Or, putting

$$\frac{2\pi}{\lambda}\left(\frac{hx^3}{3a^2}\cos\theta - x\sin\theta\right) = \delta$$

$$V = k\int \sin\left(2\pi\frac{t}{T} - \delta\right) dx$$

$$= k\int \cos\delta\, dx \sin 2\pi\frac{t}{T} - k\int \sin\delta\, dx \cos 2\pi\frac{t}{T}.$$

Since $\cos\delta dx$ and $\sin\delta dx$ appear in this equation as amplitudes it follows, if

$$A = k\int \cos\delta\, dx$$

and

$$B = k\int \sin\delta\, dx$$

that the resultant amplitude $C = \sqrt{A^2 + B^2}$.

But to find A and B it is necessary to integrate the given expressions over the effective portion of the wave front, or through limits that produce essentially the same results. Such limits may be determined as follows:

Let x and x_1 be so situated that the difference between their distances from P is $\frac{\lambda}{2}$, and their combined result at that point, therefore, zero. That is, let

$$\frac{h}{3a^2}(x^3_1 - x^3)\cos\theta - (x_1 - x)\sin\theta = \frac{\lambda}{2}$$

or

$$x_1 - x = \frac{\lambda}{2\cos\theta}\cdot\frac{3a^2}{h(x_1^2 + x\,x_1 + x^2) - \tan\theta}$$

Considering the primary bow in which h has its least value, 4.89, nearly, for $\mu = \frac{4}{3}$; assuming the radius, a, of a drop to be 1 mm., and writing δx for $x_1 - x$, it follows, since θ is small, that

$$x^2\delta x = .0002 \text{ mm}^3, \text{ roughly, for yellow light.}$$

Hence, δx decreases rapidly with increase of x (even when $x = .1$ mm., $\delta x = .02$ mm.), and the successive portions of the curve beyond a very short distance from the inversion point, O (Fig. 144), completely neutralize each other, and, therefore, no error will be introduced by integrating between infinity limits instead of between the *unknown* limits of the effective portion of the wave front.

Hence,

$$A = k\int_{-\infty}^{+\infty}\cos\delta\,dx$$

and

$$B = k\int_{-\infty}^{+\infty}\sin\delta\,dx$$

But as the sign of B reverses with that of x, while that of A remains unchanged,

$$B = 0$$

and

$$A = 2k\int_0^{\infty}\cos\frac{2\pi}{\lambda}\left(\frac{hx_3}{3a^3}\cos\theta - x\sin\theta\right)dx.$$

Putting

$$\frac{2h}{3a^2\lambda}x^3\cos\theta = \frac{u^3}{2},$$

from which

$$dx = \left(\frac{3}{4h}\frac{a^2\lambda}{\cos\theta}\right)^{1/3} du,$$

and

$$\frac{2x}{\lambda}\sin\theta = \frac{zu}{2}$$

$$A = 2k\int_0^\infty \left(\frac{3a^2\lambda}{4h\cos\theta}\right)^{1/3} \cos\frac{\pi}{2}(u^3 - zu)\, du,$$

which is Airy's rainbow integral, in which

$$\frac{\pi}{2}(u^3 - zu)\, du = \delta,$$

the difference in phase.

Putting

$$\int_0^\infty \cos\frac{\pi}{2}(u^3 - zu)\, du = f(z)$$

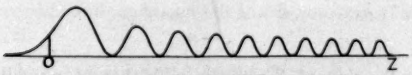
$$A = 2k\left(\frac{3a^2\lambda}{4h\cos\theta}\right)^{1/3} f(z) = M f(z), \text{ say,}$$

and the intensity,

$$A^2 = M^2 f^2(z).$$

The evaluation of $f(z)$ is not simple, but it has been accomplished through mechanical quadrature by Airy [180] and through development in series by both Airy [181] and Stokes.[182]

FIG. 145.



Periodic changes of intensity of monochromatic light.

The following table and its graph (Fig. 145), both from Pernter-Exner's "Meteorologische Optik," give certain values of z and $f^2(z)$, corresponding to monochromatic light of a particular wave-length and drops of a definite size.

[180] *Cambridge Phil. Trans.*, vi, p. 379.
[181] *Cambridge Phil. Trans.*, viii, p. 595.
[182] *Cambridge Phil. Trans.*, ix, part i.

Values of $F^2(Z)$ for Different Values of Z.

z	$f^2(z)$	z	$f^2(z)$	z	$f^2(z)$
−2.0	0.006	3.4	0.609	8.8	0.189
−1.8	0.011	3.6	0.586	9.0	0.373
−1.6	0.018	3.8	0.436	9.2	0.320
−1.4	0.030	4.0	0.225	9.4	0.100
−1.2	0.048	4.2	0.051	9.6	0.001
−1.0	0.074	4.4	0.003	9.8	0.054
−0.8	0.113	4.6	0.104	10.0	0.240
−0.6	0.168	4.8	0.297	10.2	0.360
−0.4	0.239	5.0	0.465	10.4	0.220
−0.2	0.331	5.2	0.501	10.6	0.022
0.0	0.443	5.4	0.379	10.8	0.013
0.2	0.571	5.6	0.172	11.0	0.170
0.4	0.706	5.8	0.014	11.2	0.338
0.6	0.836	6.0	0.022	11.4	0.270
0.8	0.941	6.2	0.174	11.6	0.050
1.0	1.001	6.4	0.370	11.8	0.004
1.2	0.996	6.6	0.450	12.0	0.140
1.4	0.914	6.8	0.353	12.2	0.320
1.6	0.758	7.0	0.141	12.4	0.256
1.8	0.547	7.2	0.010	12.6	0.045
2.0	0.319	7.4	0.046	12.8	0.006
2.2	0.125	7.6	0.230	13.0	0.136
2.4	0.014	7.8	0.394	13.2	0.314
2.6	0.016	8.0	0.363	13.4	0.202
2.8	0.131	8.2	0.150	13.6	0.013
3.0	0.317	8.4	0.010	....	.....
3.2	0.502	8.6	0.038	....	.....

Maxima and Minima.

Maxima	z	$f^2(z)$	Minima	z
1.	1.0845	1.005	1.	2.4955
2.	3.4669	0.615	2.	4.3631
3.	5.1446	0.510	3.	5.8922
4.	6.5782	0.450	4.	7.2436
5.	7.8685	0.412	5.	8.4788
6.	9.0599	0.384	6.	9.6300
7.	10.1774	0.362	7.	10.7161
8.	11.2364	0.345	8.	11.7496
9.	12.2475	0.330	9.	12.7395
10.	13.2185	0.318	10.	13.6924

It will be noticed that the first maximum does not coincide with $z = 0$, nor, therefore, with $\theta = 0$, the direction of the ray of minimum deviation; that the intensity of the first maximum, corresponding to a principal bow, is much the greatest; and that the succeeding maxima, corresponding to the supernumerary

bows, and also the angular intervals between successive maxima, continuously decrease, at a decreasing rate, with the increase of θ.

Distribution of Colors in the Rainbow.—The above discussion of the distribution of light intensity applies, as stated, to monochromatic light. When the source of light simultaneously emits radiations of various wave-lengths, as does the sun, a corresponding number of bows, each consisting of a sequence of maxima and minima, are partially superimposed on each other. In this way different colors are mixed, and thus the familiar polychromatic rainbow produced.

The particular mixing of colors that obtains is the result of several coöperating causes. Thus the distribution of intensity, as illustrated by Fig. 145, depends on phase difference, as given by the expression,

$$2\pi \frac{\frac{h}{3a^2} x^3 \cos\theta - x \sin\theta}{\lambda}.$$

The angular intervals between maxima, say, increase, therefore, with λ, and, consequently, coincident distribution of the intensities of any two colors is impossible. Again, since the direction of the ray of minimum deviation varies with the refractive index, as already explained, and that in turn with the wave-length or color, it follows that the direction of the zero point on the intensity curve, near which the first maximum lies, correspondingly varies. Obviously, then, these two causes together produce all sorts of color mixings that in turn arouse widely varied sensations.

To determine, however, just what color mixtures are induced by drops of any given size, it obviously is necessary to express the values of the abscissa, z, of the intensity curve (Fig. 145) in terms of angular deviation from the corresponding principal ray, since the direction of each such ray fixes the position of origin of its particular curve.

Let, then, as before,

$$\frac{z u}{2} = \frac{2 x}{\lambda} \sin\theta$$

or

$$z^3 = \left(\frac{4 \sin\theta}{\lambda} \frac{x}{u}\right)^3$$

Also, as before, let

$$\left(\frac{x}{u}\right)^3 = \frac{3a^2 \lambda}{4 h \cos\theta}$$

hence,

$$z^3 = \frac{48\ a^3}{h\ \lambda^3} \sin^2\theta \tan\theta .$$

But whatever the value of θ from 0° to 30°

$$\frac{\sin^2\theta \tan\theta}{\theta^3} = 1, \text{ to within } .0055.$$

Hence, approximately,

$$z^3 = \frac{48\ a^3}{h\ \lambda^3}\ \theta^3,$$

or

$$\theta = \frac{z}{2a^3}\left(\frac{h\ \lambda^3}{6}\right)^{1/3}$$

From this equation it appears that the angular distance between any two successive intensity maxima varies directly as the cube root of the square of the wave-length and inversely as the cube root of the square of the diameter, or other linear dimension of the parent drop. That is, this interval is greater for red light than for blue, and greater for small drops than for large ones.

The following table, copied with slight changes, from Pernter-Exner's "Meteorologische Optik," gives the values of θ in minutes of arc per 0.2 z, for lights of different wave-length and drops of different size.

Angle in Minutes per 0.2 z, Primary Bow.

a = in microns	5	10	15	20	25	30	40	50	100	150	250	500	1000
λ, in microns	Angle in minutes												
.687	85.8	54.0	41.2	34.0	29.3	26.0	21.4	18.5	11.7	8.9	6.32	3.98	2.51
.656	83.0	52.3	39.9	32.9	28.4	25.1	20.7	18.0	11.0	8.6	6.10	3.84	2.43
.589	77.0	48.5	37.0	30.5	26.4	23.3	19.3	16.6	10.5	7.9	5.67	3.57	2.26
.527	71.2	44.9	34.2	28.2	24.4	21.6	17.8	15.4	9.6	7.4	5.25	3.31	2.10
.494	68.1	42.8	32.6	27.0	23.1	20.6	17.0	14.7	9.3	7.1	5.02	3.15	2.03
.486	67.2	42.3	32.3	26.7	22.8	20.3	16.8	14.5	9.1	7.0	4.94	3.12	1.99
.449	63.4	40.0	30.5	25.2	21.7	19.2	15.9	13.7	8.6	6.6	4.67	2.93	1.88
.431	51.5	38.8	29.6	24.4	21.0	18.6	15.4	13.3	8.3	6.4	4.53	2.87	1.82

By the aid of this table; a table of intensity distribution, $M^2f^2(z)$, along the coördinate z; and the following relative intensities,

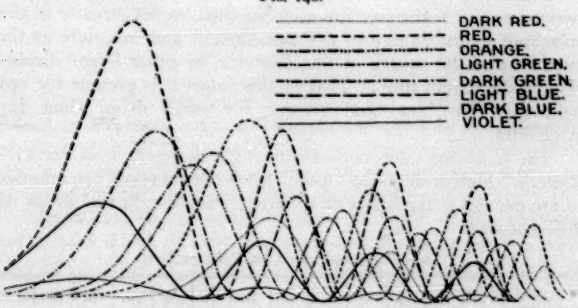
λ	.687	.656	.589	.527	.494	.486	.449	.431
I	20	86	250	152	121	134	163	74

Pernter has constructed Figs. 146 and 147 that show the intensity distributions of these several colors due to drops of .5 mm. and .05 mm. radius, respectively.

It still remains to determine the color at any particular point at which the relative intensities of the several colors are known. This can be done by the aid of Maxwell's [183] color triangle, as explained in detail by Pernter.[184]

Relation of size of Drop and Wave-length to Intensity.— Since the Airy expression for the amplitude of the vibration produced at a distant point by the effective portion of the emitted wave front involves the factor, $(\lambda a^2)^{\frac{1}{6}}$, it is evident that the cor-

FIG. 146.



Distributions of colors by drops of 0.5 mm. radius.

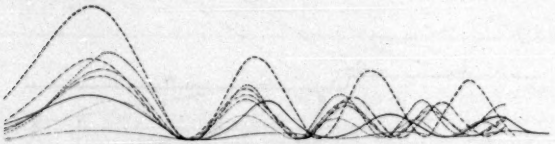
responding intensity, which is proportional to the square of the amplitude, will be proportional to $(\lambda a^2)^{\frac{1}{3}}$. This, however, is based on the assumption that the effective light from the drop comes strictly from the *line* of a great circle. As a matter of fact, it actually comes from a narrow belt whose effective angular width, as measured from the centre of the drop, is inversely proportional to the curvature, or directly to the radius a, and inversely proportional to the wave-length.[185] Hence, the actual intensity is proportional to a.

[183] *Trans. Roy. Soc.*, p. 57, 1860.
[184] *Meteorologische Optik.*
[185] Mascart, C. R., V. 115, p. 453, 1892.

A larger fraction of the short wave-length light is effective, therefore, than of the long. Further, the rainbow bands produced by very small droplets are not only broad, as previously explained, but also feeble, and as their colors necessarily are faint they frequently are not distinguished—the bow appearing as a mere white band.

Popular Questions About the Rainbow.—A few popular questions about the rainbow need perhaps to be answered. "What is the rainbow's distance?" In the sense of its proximate origin, the drops that produce it, it is nearby or far away, according to their respective distances, and thus extends from the closest to the farthest illuminated drops along the elements of the rainbow cone. Indeed, the rainbow may be regarded as consisting of coaxial, hollow conical beams of light of different colors seen edgewise from the vertex, and thus having great depth or extent in the line of sight.

FIG. 147.



Distributions of colors by drops of 0.05 mm. radius.

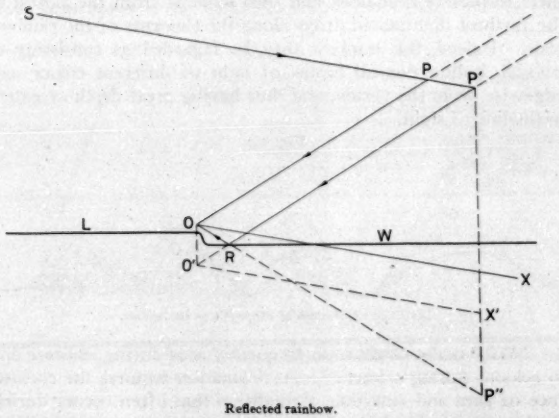
"Why is the rainbow so frequently seen during summer and so seldom during winter?" Its formation requires the coexistence of rain and sunshine, a condition that often occurs during local convectional showers but rarely during a general cyclonic storm, and as the former are characteristic of summer and the latter of winter, it follows that the occurrence of the rainbow correspondingly varies with the seasons.

"Why are rainbows so rarely seen at noon?" As above explained, the centre of the rainbow's circle is angularly as far below the level of the observer as the sun is above it, hence no portion of the bow can be seen (except from an elevation) when its angular radius is less than the elevation of the sun above the horizon. Now, during summer, the rainbow season, the elevation of the sun at noon is nearly everywhere greater than 42°, the angular radius of the primary bow, or even 51°, the radius of the

secondary bow. A rainbow at noon, therefore, is, except for very high latitudes, an impossible summer phenomenon, and, of course, a rare winter one, for reasons given above, even where possible.

"Do two people ever see the same rainbow?" Theory teaches, and ordinary experience shows, that as the observer remains stationary or moves, so also, other things being equal, does his rainbow. If then, two observers initially close together should move in opposite directions, each would find his rainbow responding in the same sense as his shadow, and presently the

FIG. 148.



Reflected rainbow.

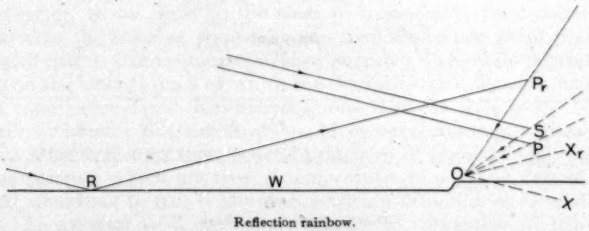
positions and, therefore, the identity of the two bows would become unquestionably different, from which it follows that as the eyes of two observers must always be separated by a greater or less distance their bows must also be correspondingly separated and different—different in the sense that they have different positions and are produced by different drops. In short, since the rainbow is a special distribution of colors (produced in a particular way) with reference to a definite point—the eye of the observer—and as no single distribution (other than uniform and infinite) can be the same for two separate points, it follows that two observers do not and cannot see the same rainbow.

"Can one see the same rainbow by reflection that he sees directly?" An object seen by reflection in a plane surface is seen by the same rays that but for the mirror would have focussed to a point on a line normal to it from the eye and as far back of it as the eye is in front. But, as just explained, the bows appropriate to two different points are produced by different drops; hence, a bow seen by reflection is not the same as the one seen directly.

Reflected Rainbows.—Since rainbows occasionally are seen reflected in smooth bodies of water they deserve, perhaps, a somewhat fuller explanation than that just given.

Let an observer be at O (Fig. 148). Under proper conditions of rain and sunshine he will see directly a primary bow due to drops on the surface of a cone formed by rotating OP about

FIG. 149.



Reflection rainbow.

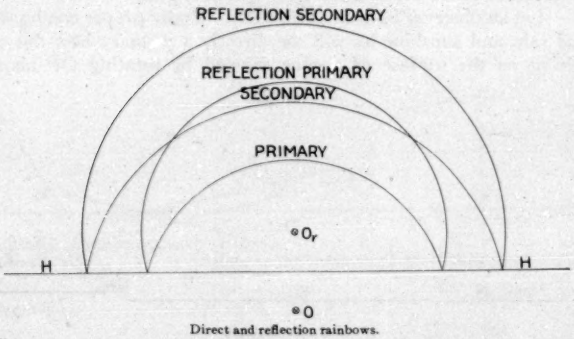
OX, parallel to SP, keeping the angle, POX, roughly 42°, constant; and by reflection in the surface of the water, W, another primary bow due to drops on the surface of a different cone, one formed by rotating $O'P'$ about $O'X'$, also parallel to SP, keeping the angle $P'O'X'$ constant. The bow seen by reflection necessarily will appear upside down, P' at P'', etc. The arc of the bow seen by reflection obviously (from the figure) will be less than that of the bow seen directly, and for that reason is likely to appear flatter.

Reflection Rainbows.—A reflection rainbow is here defined as one due to reflection of the light source, the sun, usually, but itself seen directly.

Let the observer be at O (Fig. 149) with the sun and a smooth surface of water, W, at his back and illuminated rain in front. The direct sunshine will give a primary and a secondary bow

along the elements of cones formed by rotating OP and OS, respectively, about the common axis, OX, parallel to the incident rays, keeping the angles POX and SOX constant; while the reflected light will give a primary bow along the elements of a cone formed by the rotation of OP_r about the axis OX_r (OX_r parallel to RP_r), keeping the angle P_rOX_r constant, and equal to POX. Reflection bows of higher orders than that of the primary are likely to be too faint to be seen.

FIG. 150.



Direct and reflection rainbows.

The angular elevation of the centre (axis really) of the reflection bows clearly is equal to the angular depression of the centre of the direct bows. Hence, the direct and the reflection bows of the same order intersect on the observer's level, as shown in Fig. 150.

(*To be Continued.*)

REFRACTORY MATERIALS AND HIGH TEMPERATURE MEASUREMENTS.*

BY

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THE TEMPERATURE SCALE.

BEFORE we consider the measurement of high temperatures, it is important to know just what is meant by temperature. There are a number of physical terms, including "energy," "force," "mass," and "temperature," the meanings of which are generally well enough understood for most purposes, but the precise definition of which is not easy. People sometimes think of temperature as being defined by the expansion of mercury in a thermometer, as by dividing the stem of a mercury thermometer between the freezing point of water and the boiling point into equal spaces. Temperatures defined in such a way would depend upon the kind of glass of which the thermometer was made, and if some other liquid than mercury were used the scale would be still different. The definition would be very arbitrary. There are some very important general principles of physics involving temperature which are true if temperature is properly defined, but would not be true if any such arbitrary definition were used.

If we start with the merely qualitative conception of temperature that we obtain through our senses, we may form a satisfactory quantitative definition if we employ the observation that, in the words of Clausius, "it is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body to another at a higher temperature." By the employment of this principle and that of the conservation of energy, it has been demonstrated that in any reversible cyclic process operating between, T_1, and a lower temperature, T_2, the ratio $\frac{Q_1}{Q_2}$ of the quantity of heat absorbed at the higher temperature to that given out at the lower depends only upon T_1 and T_2. Lord Kelvin has proposed that absolute temperature be so defined that

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

* Presented at a meeting of the Section of Physics and Chemistry held Tuesday, February 14, 1918.

This is a single equation involving two temperatures, and to make the definition complete, we specify arbitrarily that the difference between the boiling point of water under a pressure of one standard atmosphere and the freezing point shall be 100°. This gives us the absolute temperature in centigrade degrees, while the ordinary centigrade scale in which the freezing point is taken arbitrarily as zero differs from the absolute scale by the difference between the absolute zero and the freezing point, which is 273° C.

The definition here presented employs the conception of a reversible thermodynamic process. The restriction to a "reversible" process is important. By this is meant a process the direction of which can be reversed by infinitesimal changes in the applied forces. For example, if a gas were compressed in a cylinder and there were no transfer of heat to or from the gas and friction could be eliminated, the force applied to the piston to produce slow compression would need to be only infinitesimally more than the resisting force exerted by the gas at that particular stage in the compression, and an infinitesimal diminution in applied force would be sufficient to reverse the action at that stage. Practically, we cannot entirely eliminate the friction of the piston, nor perfectly control the transfer of heat, but this results merely from defects in our machine, and does not depend upon properties of the gas, and by making allowance for these imperfections we can learn the influence of a reversible process upon the gas.

An example of an irreversible process is presented by a gas mixture in which combustion is taking place. Our definition of temperature does not apply to a flame; a flame has no temperature. This fact, which is not always realized, illustrates the importance of the careful consideration of definitions, and is one reason why I have discussed the definition here. Perhaps some other definition of temperature could be formed which would be applicable to a flame, but I am not aware that any consistent one has been developed. We may think of the temperature of the flame as the temperature which a solid object placed in it will acquire; but this temperature depends upon various conditions, including the loss of heat from the solid by conduction and radiation, and even if we can prevent such loss of heat it depends upon the nature of the solid, for the solid may have a marked

influence upon the rate of combustion, and probably has some influence in any case. When a flame contains solid particles of carbon, as a yellow gas flame does, these solid particles have a temperature and it can be measured, but it does not represent the temperature of the gaseous part of the flame. For practical purposes, we do not want to know the real temperatures of flames, but only that which they will impart to solids or liquids, and this, of course, can be determined, but it depends upon conditions.

The definition of temperature which has been presented, which may be called the thermodynamic definition, is equivalent to that in terms of the expansion of an ideal gas, that is, one in which the product of pressure and volume is proportional to absolute temperature, and which does not change temperature when it expands without doing work. No such gas exists, but if we use a real gas, a determination of certain physical constants of the gas and the use of the laws of thermodynamics enable us to calculate the difference between the apparent temperature indicated by the expansion of the gas and the true thermodynamic temperature.

The International Committee on Weights and Measures adopted as the standard thermometer scale that defined by the constant-volume hydrogen thermometer. Since hydrogen approaches closely the condition of a perfect gas, the difference between this scale and the thermodynamic scale is very small.

Gas thermometers are of value as primary standards, but their use requires a great deal of care, and they are not well adapted to routine work. At high temperatures they are especially limited by the circumstance that there are no materials from which the thermometer bulbs can be made which resist temperatures above certain limits without permitting the gas to leak out, and without undergoing permanent deformation. At high temperatures nitrogen has been used in preference to hydrogen because nitrogen does not leak out as fast. Nitrogen thermometers have been used at temperatures up to about 1600° centigrade. They are used for the calibration of pyrometers of other types, and especially for the determination of standard temperatures, most of them the melting points of pure substances, which temperatures can be readily reproduced and serve as practical standards.

THERMOCOUPLES.

Of the pyrometers used practically, thermocouples are the most numerous. A thermocouple consists essentially of two wires of different materials joined end to end. If the junction is heated, a small electromotive force is produced, which may be measured by an electrical instrument attached to the other ends of the wires. The electromotive force depends not only upon the temperature of the hot junction, but also upon the temperatures of the other junctions in the circuit, and either the cold junctions must be kept at constant temperature, or their temperatures must be measured and a suitable allowance made. The metals of the couple must, of course, be able to withstand the temperatures to be measured, and for accurate work they must not change under the action of heat, and they must be uniform. For high temperatures the best couples known are those having one wire made of pure platinum, the other of an alloy of platinum with about 10 per cent. of rhodium. They are, of course, rather expensive. The chief difficulty in their use arises from the necessity of protecting the platinum very well from contamination, which causes a change in electromotive force and an error in reading. Any metal, even metal vapor, is injurious. A reducing atmosphere is also very injurious, perhaps because it introduces silicon into the platinum by reduction of some of the silica compounds usually present in the furnace used. The platinum couples are usually protected by air-tight procelain tubes.

When the temperature to be measured is not very high, cheaper couples made of base metals may be used. For instance, for low temperatures couples made of copper and constantan are useful, constantan being an alloy of copper and nickel; for higher temperatures iron and constantan, and various alloys containing nickel, iron, chromium and aluminum are used. Base metal couples cannot well be used above about 1100° or 1200° C.

The electromotive force is commonly measured by a galvanometer, the scale of which may be graduated to read temperature directly. The galvanometer should have a high resistance, else its indication will depend too much upon the resistance of the couple, which depends upon how much of the couple is heated. For the most accurate work a potentiometer is employed.

Before a thermocouple is used a calibration must usually be made to find the relation between the electromotive force of the

couple and the temperature. This cannot be calculated from any known theoretical relations. If several couples are made from a stock of wire drawn from the same ingot, one calibration on that lot of wire may be sufficient for practical purposes, but in the case of most metals it is difficult to obtain wire that is sufficiently uniform so that couples made from different portions of it by joining to uniform wire of another material will give the same electromotive force at the same temperature.

Couples may be calibrated by means of known melting points or boiling points. The melting points of copper, silver, antimony, zinc and lead are commonly employed. The metal is melted in a crucible of some material that will not contaminate it, the couple is placed inside a protecting tube and immersed deep in the metal, the metal is allowed to cool slowly and the electromotive forces measured at definite times are observed. While the metal is freezing the electromotive force remains nearly stationary and the value then observed corresponds to the melting point of the metal.

RESISTANCE THERMOMETERS.

This form of pyrometer depends upon the change of resistance of a metal with temperature. The metal used is almost always platinum. They are not used to quite so high temperatures as platinum thermocouples, partly because to have enough resistance, the wire must be rather fine, and is therefore more easily influenced by contamination. They are capable of detecting very small changes of temperature, which is important for special purposes.

The current which measures the resistance of the platinum must be carried to it by leads, and it is necessary to eliminate the resistance of the leads from the resistance measured, since this resistance depends upon the distribution of temperature along the leads. This resistance may be eliminated by either of two methods, known as the three-lead method, and the four-lead method. In the three-lead method, two leads are used on one side of the platinum resistance, and if the leads are in all respects the same, and if the resistance of the two in series is subtracted from the resistance through the platinum with a lead at each end, the result is the resistance of the platinum. The four-lead method is the most reliable for accurate work. In this method two leads carry a current through the platinum resistance, while the other

two serve for the measurement of the difference in potential between the two ends of the resistance.

The platinum wire is commonly wound on mica. It should be wound in such a way that the inequality in the expansion of platinum and the supporting material does not put a strain on the platinum, for this would change its resistance.

Resistance thermometers, like thermocouples, must be calibrated against known temperatures. The boiling points of sulphur, naphthalene and water, and the melting point of ice are commonly used.

OPTICAL AND RADIATION PYROMETERS.

The pyrometers so far described all have parts that must be put into the place where the temperature is to be measured. When we need to measure extremely high temperatures, say temperatures above 1500° C., we cannot find materials suitable for the purpose that are sufficiently permanent at high temperatures, but we can then determine the temperatures by a study of the light and heat radiation emitted.

The intensities of the radiations of various wave lengths emitted depend not only upon the temperature but also upon the material that emits the radiation. But fortunately there are certain conditions that give a kind of radiation which depends upon the temperature only. If a closed vessel of any solid material is heated to a uniform temperature the nature of the radiation inside the vessel depends only upon the temperature and not upon the material. Such a vessel is commonly called a "black body." If a hole is made in the wall of the vessel that is small relative to the dimensions of the vessel, the radiation within the vessel is changed only a little, and the radiation that comes out of the hole is very nearly "black body radiation." This radiation is of all wave lengths, both visible and invisible. The influence of temperature upon its composition is known, and is expressed by the following equations:

$$E = \sigma T^4 \qquad \text{Stefan and Boltzmann}$$

$$I = \frac{C_1}{\lambda^5 \left(e^{\frac{C_2}{\lambda T}} - 1 \right)} \qquad \text{Planck}$$

$$I = \frac{C_1}{\lambda^5 e^{\frac{C_2}{\lambda T}}} \qquad \text{Wien}$$

E = total energy radiated from unit area in unit time.
λ = wave length.
I = energy per unit range of wave length radiated from unit area in unit time.
T = absolute thermodynamic temperature.
e = base of Napierian logarithms.
σ, C_1, C_2 = universal constants.

When the wave length is expressed in centimetres, and the area in square centimetres, and the gram calorie, second and centigrade degree are used, the values of these constants are:

$$\sigma = 1.28 \times 10^{-12}$$
$$C_1 = 8.8 \times 10^{-23}$$
$$C_2 = 1.445$$

The Stefan-Boltzmann law is strictly correct. It was obtained experimentally by Stefan and was later proved on the basis of thermodynamics by Boltzmann.

The Planck law agrees with observation, and Planck and others have presented theoretical demonstrations. These demonstrations have not been founded upon such unquestionable principles as those underlying the Stefan-Boltzmann law, and have given rise to much discussion among physicists; but the weight of observational and theoretical evidence is such that the relation may be safely employed.

The Wien law, which was published before the Planck law, is not strictly correct; but its errors are large only for long wave lengths in the infra-red radiation, while for visible light the difference between this law and Planck's is below the limits of observation. It is, therefore, employed in optical pyrometry, and leads to much simpler calculations than the Planck law.

Each of these laws gives the radiation from the black body. There is also radiation from the surroundings or from the observing instrument to the black body, and the net transfer of energy is the difference between the two; but in pyrometry the latter effect is negligible.

There are two important classes of pyrometers employed at very high temperatures; those that depend upon the intensity of visible light, called optical pyrometers, and those that depend upon the total radiant energy of all wave lengths, commonly called radiation pyrometers.

The Holborn-Kurlbaum or Morse optical pyrometer has been employed in the most accurate measurements of very high temperatures. It consists essentially of a telescope, which may have little or no magnifying power, with a small filament lamp in the focal plane of the eye-piece, which is covered with a piece of red glass of a kind transmitting as nearly as possible monochromatic light. Upon looking into the instrument one sees an image of the object upon which it is sighted, and over this image the lamp filament. The current in the filament is varied by means of a variable resistance until the end of the filament has the same brightness as the field, when it becomes nearly or quite invisible. The current is then measured. The instrument is calibrated by observing known temperatures and forming a curve or equation giving the relation between current and temperature. The lamp must not be heated to as high a temperature as that at which it would be operated for illuminating purposes, because at such a temperature its temperature-current relation would change with time. When it is necessary to measure higher temperatures, a black absorption glass is placed between the instrument and the heated object, and the apparent temperature observed through the glass is measured. The glass may be calibrated by observing through it an object at a temperature low enough to be also observed directly. Then, if

T_1 = known temperature
T_2 = unknown temperature
T_3, T_4 = apparent temperatures when T_1 and T_2, respectively, are observed through the glass,

all temperatures being on the absolute scale, the unknown temperature can be calculated from the relation

$$\frac{1}{T_4} - \frac{1}{T_2} = \frac{1}{T_3} - \frac{1}{T_1},$$

which can be deduced from the Wien law. The quantity $\frac{1}{T_3} - \frac{1}{T_1}$ is a constant characteristic of the absorption glass for the wave length used. No knowledge of the radiation constants or the wave length is required when this method is employed. By using a sufficiently dark glass it is possible to measure the highest temperature that can be produced.

In the Wanner of Scimatco optical pyrometer the field is divided into two halves, each of which is uniformly illuminated, one by the object upon which the instrument is sighted, the other

by an electric lamp, which is operated at a constant brightness by occasional comparison with an amyl acetate flame. A system of prisms permits the transmission only of light in a portion of the red end of the visible spectrum and the employment of two polarizing prisms, one fixed, the other movable, permits the variation of the intensities of the two parts of the field until they match. The angle to which the movable prism has been turned indicates the temperature.

There are a number of optical pyrometers in which the field is divided into two parts, one illuminated by the heated object, the other by a standard source of light, the two parts being brought to the same brightness by placing some variable absorbing device in the path of the more intense source of light.

In radiation pyrometers, the radiant heat is caused to fall upon one junction of a thermocouple, which is connected to a galvanometer. In the Féry type the radiation is converged by a concave mirror onto a thermocouple exposed under an opening in a small shielding box. The radiation entering is limited by this opening. In the Thwing type, the radiation is limited by a diaphragm at the front of the instrument. Radiation pyrometers give good results with suitable precautions, but they are somewhat influenced, especially in the Féry type, by stray heat, which may reach the thermocouple otherwise than by direct radiation.

Both optical and radiation pyrometers are usually calibrated to read correctly when sighted upon a black body. The interior of a furnace fairly uniformly heated usually approximates black body conditions, but it is often required to measure the temperature of objects in the open, for example, red-hot billets of metal in the process of rolling. In such cases it is necessary to know the emissivity of the particular material observed, that is, the ratio of the intensity of the radiation it emits to that emitted by a black body, or to calibrate the pyrometer by comparing observations made with it with the temperature of the material as measured in some other way, as by thermocouples. The emissivity is a function of both temperature and wave length.

PYROMETRIC CONES.

Pyrometric "cones" are employed for temperature control in many industries. They consist of slender pyramidal molded forms of various materials, which are set up in a slightly inclined

position in a base of clay or other material, and inserted in the furnace the temperature of which is to be controlled. When a cone has softened sufficiently to bend over until its tip touches the support, the temperature corresponding to that cone is considered to have been reached. A series of cones corresponding to temperatures at intervals of about 20° C. is employed.

Cones are not adapted to the accurate measurement of temperature, since their falling over depends not only upon the temperature but also upon the length of time of heating. It is, therefore, not advisable to attempt to correlate definite temperatures with the cone numbers. However, they are very useful in the control of many industrial operations when they are used under nearly constant conditions and when it has been found by trial that the operation of a furnace at a certain cone number gives satisfactory results.

TEMPERATURE TABLE.

Some of the more important or more interesting high temperatures are shown in the table. The very high temperatures, except those of the stars, have been measured with optical or radiation pyrometers. The determination of the surface temperature of the sun is based upon the assumption that it radiates like a black body. The estimates of the surface temperatures of the stars have been based upon observations of the distribution of energy in their spectra. These estimates are quite uncertain. The temperatures in bold-face type are accurately known and are employed as fixed points in the calibration of pyrometers.

	Centigrade Degrees
Stars (at surface)	up to about 20,000
Sun (at surface)	6,000
Highest temperature yet produced artificially	6,000
Carbon arc	3,800
Tungsten melts	3,400
Magnesia melts	2,800
Lime melts	2,570
Iron boils	2,450
Copper boils	2,300
Alumina melts	2,050
Platinum melts	1,755
Fireclay melts	1,560 to 1,725
Palladium melts	1,550
Pure iron melts	1,530

	Centigrade Degrees
Nickel melts	1,452
Cast iron melts	1,100 to 1,250
Copper melts	1,083
Gold melts	1,063
Silver melts	961
Antimony melts	630
Sulphur boils	444.6
Zinc melts	419
Tin melts	232

MELTING POINTS.

One of the most important properties of a refractory material is its melting point. We are accustomed to thinking of a melting point as a temperature at which a substance changes from a condition in which it does not flow perceptibly under its own weight to a condition in which it flows readily. Most pure substances have melting points that are quite definite and unmistakable, but with some materials this is not the case. For example, when glass is heated, the transition from its ordinary condition to a distinctly fluid condition is a very gradual one and occupies a range of several hundred degrees. Some other substances which possess perfectly definite temperatures of transition to a fluid phase, undergo changes resembling fusion at lower temperatures. Solids in general become softer when heated; they may become so soft as to yield to their own surface tension, with the result that sharp corners are rounded off. Rutile, in its natural slightly impure condition, melts at about 1700° C. The melting point is very definite, there being a sharp transition from a rather soft solid to a liquid of low viscosity, with a considerable absorption of heat. Yet at temperatures 100 degrees or more below the melting point, corners are rounded off and small particles become sintered together. The sintering of clay is probably a similar phenomenon of surface tension. Clay exists generally in the form of extremely fine particles, its unusually slight solubility in water accounting for this condition. Fine particles are more readily united by surface tension than large ones.

A melting point can be precisely defined, in such a manner that it corresponds to a real discontinuity in the nature of the substance, only as the temperature at which a crystalline or anisotropic phase and an amorphous or isotropic phase of the same composition can exist in contact in equilibrium.

While this definition is satisfactory for the case of a pure substance, so complex a mixture as an ordinary fire brick usually has no single definite melting point according to this definition, since several anisotropic phases may be present, all differing in composition from the isotropic phase produced by fusion. We can then only select the temperature at which the transition from a rigid to a fluid state seems most distinct, and can call this the melting point only by apology. In the case of fire bricks, the transition temperatures so found are fortunately sufficiently definite to make their determination of practical value.

Some substances after being melted and then cooled below the melting point, crystallize so slowly that cooling can be continued down to ordinary temperature without noticeable crystallization, the material usually becoming so viscous that it is for practical purposes a solid, though in consistency with our definition of melting point, it may be considered a liquid. Ordinary glass and silica glass belong in this class.

REFRACTORY MATERIALS.

CARBON.

Carbon exists in the allotropic crystalline forms of graphite and diamond, and in the amorphous state. Diamond and amorphous carbon are converted to graphite by the action of very high temperature.

Graphite is the most refractory material known. At the temperature of the carbon arc, about 3800° C., its vapor pressure becomes equal approximately to that of the atmosphere, and it vaporizes without melting. It is very doubtful whether it has ever been melted. At very high temperatures phenomena sometimes take place which give it very much the appearance of having been melted. For instance, if an arc is produced in a space containing no oxygen, the carbon vaporizes from the part heated by the arc, and the vapor condenses on adjoining parts, often forming nodular masses, which look very much as though they had been produced by fusion.

Graphite is a very useful refractory material for laboratory purposes and for many industrial purposes, for the reasons that it is very refractory, conducts electricity, and can be easily machined into almost any shape. Several forms of high temperature laboratory furnaces have graphite electric heaters.

METALS.

The most refractory metals are of interest in connection with the manufacture of filament electric lamps. Of these metals tungsten is the most refractory and is the one now in use in lamps. Tantalum, molybdenum and osmium are also very refractory. Tungsten is produced in the form of powder by the reduction of its oxide. This powder can be molded into rods, crucibles and other forms, and sintered by heating to 1800° to 2000° C. The material produced in this way is very porous. Compact ductile tungsten, suitable for the drawing of wire for lamp filaments, is made by hammering the porous material, with suitable heat treatment.

Metals of the platinum group are all refractory. Platinum is used for crucibles, thermocouples, resistance thermometers, electric heaters, and other purposes. Electric furnaces with platinum heaters may be used at temperatures up to about 1500° C., though it is commonly not advisable to approach very close to this limit.

Laboratory furnaces with iridium heaters have been used in a few instances, but they are very expensive. They give temperatures up to about 2000° C. The advantage of platinum and iridium heaters over the cheaper graphite heaters is that they can be used in contact with air, while graphite heaters would burn in air and must be used in a vacuum or in a non-oxidizing gas. Graphite produces reduction or contamination in many materials which it may be desired to heat. Some base metals, especially alloys containing chromium and nickel, are used for electric heaters at moderate temperatures. A temperature of 1100° C. is about the limit for such heaters.

OXIDES.

Many metallic oxides are highly refractory. Magnesia, one of the most refractory, has a melting point of 2800° C., while that of zirconia is possibly higher. Magnesia is used as an industrial and laboratory refractory. Zirconia with yttria was used in the Nernst electric lamp, which has now gone out of use. Within the last few years natural deposits of zirconia have been found in sufficient quantity to form an industrial source. Previously it was obtained from the silicate, zircon.

Lime, with a melting point of 2570° C., is very refractory, and has some advantage in mechanical properties over magnesia. It

has the disadvantage of slaking when exposed to air. It has been used for some laboratory purposes, and also in the lime light, which has now been supplanted by the arc light.

Thoria and ceria are used in Welsbach gas mantles. Beryllia is very refractory, but has received little application.

Alumina, melting at 2050° C., forms the basis of the abrasive known as "alundum," which is made by fusing alumina and grinding the compact mass thus obtained.

Silica is used commercially as brick, and also forms a valuable glass. It melts at 1710° C., forming so viscous a liquid that very little crystallization occurs on cooling, and the product is a glass of very low expansion coefficient, which makes it capable of resisting sudden changes of temperature. Since this is made by fusing quartz, the most common crystalline form of silica, it is often called "quartz glass," though this is not correct, for after fusion it is no longer quartz.

REFRACTORY BRICK.

The most commonly employed refractory brick is that made of fire clay. The essential ingredient of clay is the compound $Al_2O_3,2SiO_2,2H_2O$. When this occurs in the pure condition it is known as kaolin. When kaolin is heated to a high temperature the resulting material is a mixture of Al_2O_3,SiO_2 and SiO_2, and has an apparent softening point of 1740° C. This is, therefore, the highest softening point obtainable in brick containing nothing more refractory than clay. Some commercial fire-clay brick approaches within a few degrees of this value.

If the material used contains in addition to clay the mineral bauxite, $Al_2O(OH)_4$, the material produced under the action of heat has an increased proportion of Al_2O_3,SiO_2, which has a melting point of 1810° C., and the brick may be more refractory than the best clay.

Silica brick are used in considerable quantity. They are made from a silica rock called "gannister," bonded with a little lime. If made of good material their melting point is very little below that of pure silica (1710° C.).

Magnesia brick of good quality are the most refractory in common use. Good brick is not readily made from pure magnesia, since its bonding qualities are poor. Commercial magnesia brick contains iron oxide, which serves as a binding material.

As the result of the presence of iron and impurities, the melting point is far below that of pure magnesia (2800° C.), but still very high, generally 2000° to 2200° C. When the brick is heated to near its melting point the vaporization of iron and other material causes the melting point to increase and renders its determination rather uncertain.

While lime is too unstable in air to be used as a refractory material on a large scale, the mineral dolomite, which consists of carbonate of lime and magnesia, gives upon firing a mixture of lime and magnesia which is more stable and is of practical use.

Chromite, FeO,Cr_2O_3, is used in the manufacture of a brick which has the valuable property of being very resistant to the fluxing action of almost any fused material. It also has a very high melting point, about 2050° C.

While the melting or softening temperature of a refractory brick is perhaps the property most often considered as an indication of its value, many other properties are of importance in various degrees, depending upon the use to be made of it. For some uses, for example in the lower tiers of blast furnaces, it must have high crushing strength, not only cold but also hot. When exposed to contact with furnace tools it must resist abrasive action. In metallurgical furnaces it must usually resist slagging action. Its thermal expansion is often important, since too much expansion may threaten the stability of furnace parts, especially roofs. If it is exposed to rapid changes of temperature or to temperatures that are not uniform, it must resist these without cracking or spalling.

LABORATORY REFRACTORIES.

In addition to the materials already mentioned, there are a number of others that are adapted to laboratory use or to other use on a small scale.

Asbestos is a familiar material of this class. Its value depends upon its heat insulating qualities and upon the fact that it can be made into sheets, fabric and cord. The kind used most in the United States is a variety of the mineral serpentine. This material has a melting point about as high as that of fire clay, but since it becomes brittle and crumbly at a red heat it is not often used at very high temperatures.

Mica is useful for many purposes because of its transparency,

and the ease with which it can be split into thin tough sheets. When muscovite mica is heated it begins to lose transparency and becomes brittle at about 700° C. These changes increase with temperature until at 1000° it is very friable, while at 1150° it is stronger again, as the result of sintering. It melts at about 1560°.

Porcelain is used for thermocouple tubes and insulators, spark plugs, crucibles, laboratory furnace tubes, and in a variety of ways. The range of melting points is wide; the best is about as refractory as fire clay.

Crucibles, tubes and other forms made from alundum (bonded fused alumina) and bonded silicon carbide are on the market. They are of use when a product less easily cracked than porcelain, or having a higher thermal conductivity, is required. They are usually bonded with plastic clay, and their softening temperature is then practically that of the clay.

The mineral steatite, which is a hydrated magnesium silicate, has some useful properties. In its natural state it is very soft and can be easily cut. If it is then fired it becomes quite hard. It melts at about 1525° C.

WAR PROBLEMS

The war has given rise to a number of problems relating to refractory materials, of considerable practical importance, both in the production of special military equipment and in the expansion of existing industries. The cutting off of the supply of some materials formerly obtained from Europe, notable magnesite and certain plastic clays, has caused difficulties and made it necessary to give careful attention to the possible sources of such materials in this country.

The war has required the rapid production of large numbers of optical instruments, including field-glasses, range-finders, gun-sights and periscopes. Before the war all optical glass used in the United States was imported from Europe, and this supply having been cut off, the rapid establishment of a new industry presenting considerable technical difficulties was necessary. One of the greatest of these difficulties was the production of the large clay pots in which optical glass is melted. These pots must not only be capable of carrying a great weight of melted glass without danger of cracking, but they must not be readily attacked by the melted glass nor add any coloring matter to it. This problem

has been satisfactorily solved, and the long period formerly required for the drying of such pots has been materially shortened.

The airplane spark plug presents another military problem in refractory materials. The life of a spark plug is none too long in an automobile engine, while in an airplane engine the conditions are much more severe, and the problem of finding suitable refractory insulating material has required serious attention.

Before the war, the best refractory porcelain came from Berlin. Such porcelain is used extensively in pyrometer tubes, which are employed in many important war industries. Its production in the United States has been undertaken, and ware of good quality is now obtainable.

Salt. (*U. S. Geological Survey Press Bulletin No.* 422.)—Salt is plentiful in the United States, and more of it was produced in 1918 than in any preceding year, yet the rise in its price helped a little to increase the high cost of living. The average price of salt per ton in 1914 was $2.09; the average price in 1918 was $3.72, an increase of 78 per cent. The lowest estimate of human value—the minimum human efficiency—is expressed in the old phrase "He's not worth his salt," but this cheap commodity brought its producers in the United States in 1918 nearly $27,000,000, which was $7,000,000, or 35 per cent., more than they realized in 1917.

Some producers reported that the cost of labor was nearly double that in 1917, and many had trouble in getting laborers of any kind. One producer reported that though the prices were higher in 1918 than in 1917 the profits were less.

The United States exported more than a million and a half dollars' worth of salt in 1918, most of it to Canada and to Cuba, though, strange to note, we sent nearly 23,000,000 pounds, valued at about $193,000, to far-away New Zealand.

Concentration of Sweet Cider by Refrigeration.—*The Weekly News Letter of the U. S. Department of Agriculture* for September 10, 1919 (vol. vii, No. 6, p. 3) describes a process by which sweet cider is concentrated by mechanical refrigeration. The fresh apple juice is frozen, and the frozen mass is crushed, then subjected to centrifugation. The frozen water is thereby separated from the mother liquor which contains the solid matter of the apple juice. Five gallons of juice yield one gallon of a syrupy concentrate. This concentrate has better keeping qualities than ordinary cider, and remains sweet indefinitely if kept in cold storage. By addition of water it may be restored to its original volume, condition, and flavor. J. S. H.

Delivering Mail to Steamer After It Has Sailed. (*Scientific American*, vol. cxxi, No. 8, p. 189, August 23, 1919.)—One of the contemplated uses of the airplane in peace-time is that of overtaking steamers at sea for the purpose of placing delayed mail aboard. Obviously, the speed of the average airplane makes it possible to overtake a steamer several hours after it has left the port, thereby extending the mail service time that many hours.

An experiment in delivering mail to a steamer is to be undertaken within a few days by C. J. Zimmerman, a skilled pilot, who will follow the steamer *Adriatic* two or three hours after she has sailed for England, and overtaking her will drop a mail pouch with a wooden float attachment into the sea just ahead of her bow. This experiment will be closely followed by the post-office authorities and the steamship men, in order to determine the practicability of the scheme.

It has also been suggested that steamers might carry small airplanes which, when the steamer neared port, might fly with bags of mail. If airplanes were employed to overtake the steamer, and one or more airplanes employed to make port some hours before the steamer, perhaps eight hours more might be saved in trans-Atlantic mail service. However, such a scheme would call for a considerable number of machines and pilots, and would entail a notable expense.

The Experimental Kelp-Potash Plant of the United States Department of Agriculture, was erected during the summer of 1917 and put into operation in the early fall of that year (*Jour. Indus. Eng. Chem.*).

One hundred tons of raw kelp per day are subjected to drying, destructive distillation, lixiviation, evaporation, and fractional crystallization, for the preparation of a high-grade potassium chloride.

By-products, kelp-oils, creosote, pitch, ammonia, bleaching-carbons, salt, and iodine are yielded in commercial quantities by this process. The main problem now in hand is their commercialization. It is believed that they will be made to yield sufficient revenue to enable the main product, potash salts, to be marketed successfully in competition with foreign sources.

Complete operating cost data are being tabulated covering the various details of manufacture. These, together with full specifications and designs, will be made available for the public.

The results obtained to date indicate that it will be possible to establish on kelp, as the basic raw material, a new American chemical industry of importance and usefulness to the nation.

THE LOW VISIBILITY PHASE OF PROTECTIVE COLORATION.*†

BY

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Since the glare field in the instrument is limited by a circular diaphragm inclined at an angle of 45° to the line of sight, the apparent shape of the field is elliptical, its longer axis being horizontal. The actual area included in the field at any time is represented approximately by the dotted ellipse, *R*, in Fig. 6. Since the piano wire leaders were very small in diameter, they were not visible from the point of observation, being below the limit of visibility set by the resolving power of the eye. The model, therefore, was suspended in the air without any visible means of support, and when the surrounding objects in the same plane were eliminated from the field of view by the diaphragms of the instrument the most important factors upon which the perception of distance depends were rendered inoperative. The model, therefore, when viewed through the visibility-metre had every appearance of a large boat on the horizon.

For the purpose of the experimental determination of the best value of R_2 under various weather conditions, a series of nine models was prepared, varying only in reflecting power. The lowest reflecting power was that given by the blackest matt paint obtainable and the highest that obtained by the whitest. The intermediate values were obtained by using gray paints of various reflecting powers. The reflection factors of these models were measured by use of a reflectometer, in which the angle of illumination was 45° and the observation normal to the painted surface. The paints used gave a surface, when dry, that was quite matt, but some specular reflection was present. While the reflection factor mentioned above does not coincide with the value of that term for all possible conditions, it seems to be the closest

* Communicated by Dr. C. E. K. Mees. Communication No. 80 from the Research Laboratory of the Eastman Kodak Company.

† Concluded from page 387, Vol. 188, September, 1919.

approximation to an average value of that term that it is possible to obtain. The values of reflection factor, R, for the series of models used are given below:

Model No.	R_2
F 2	.75
F 12	.60
F 3	.45
F 13	.36
F 4	.28
F 14	.20
F 5	.15
F 15	.10
F 6	.05

A typical set of observations is given in Table I.

TABLE I.

Date: November 23, 1917, 3 to 4 P.M.
Weather: Sky covered with heavy uniform clouds, light snow falling, lake rough, strong N. W. wind.
Station: 1.

Model No.	B_1	B_v	W	Log B_v	B_v	V	$\frac{dV}{dR_2}$
F 2	335	655	.37	3.72	5248	15.6	+
F 12	327	630	.37	3.55	3311	10.1	+
F 3	320	575	.40	3.16	1445	4.5	+
F 13	320	575	.40	2.47	1295	0.9	+
F 4	320	575	.40	3.34	2180	6.8	—
F 14	300	535	.40	3.45	2810	9.4	—
F 5	287	525	.39	3.57	3710	12.9	—
F 15	280	550	.37	3.67	4670	15.7	—
F 6	280	557	.36	3.74	5490	19.6	—

It will be noted that W is not quite constant but is approximately so for the entire set of observations. Model F 13 gave the lowest visibility value. The reflection factor for F 13 is .36, which is almost exactly equal to W, the weather coefficient of which the average value for the set is .38. The visibility values from this set of data are plotted, Curve 1, Fig. 8, as a function of the reflection factors of the various models.

The two elements of the curve intersect at the point $V = 0.9$, $R_2 = .36$. The value of R_2 represents the optimum value of the reflection for this particular weather condition and will be designated by the symbol O_r.

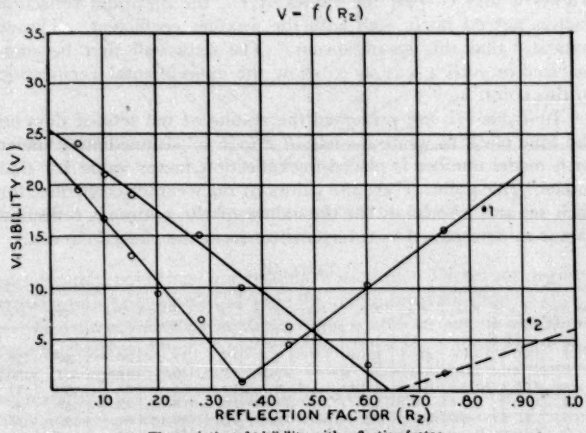
Another complete set of data is given in Table II. The value of W in this curve is much higher than in the previous case, and not constant for the entire set.

TABLE II.

Date: November 22, 1917, 2.30 to 3.30 P.M.
Weather: Cloudy, heavy cloud banks, not uniform.
Station: 1.

Model B	B	B	W	Log B_v	B_v	V	$\frac{dV}{dR_2}$
F 2	565	640	.63	2.98	955	1.7	+
F 12	590	730	.60	3.20	1590	2.7	—
F 3	640	760	.61	3.60	3990	6.2	—
F 13	710	730	.71	3.80	6610	9.3	—
F 4	710	680	.73	4.03	10800	15.	—
F 14	720	670	.76	4.90	12400	17.	—
F 5	740	630	.85	4.14	13900	19.	—
F 15	760	630	.87	4.20	15900	21.	—
F 6	760	630	.87	4.26	18200	24.	—

FIG. 8.



The variation of visibility with reflection factor.

The data from Table II. are plotted in Fig. 8, Curve 2. In this case it will be noted that for only one visibility value is the

sign of the first derivative $\frac{dV}{dR_r}$ positive, indicating that only one of the models used has a sufficently high reflection factor to give a greater brightness than that of the sky background. Since there is only one point on the element of positive slope, that element is indeterminative in position, but knowing that it should cut the element of negative slope at a point near the axis of $V = O$, its approximate position can be represented by the broken line in the figure. Thus the optimum value O_r, of the reflection factor in the case must be approximately .65, which value agrees fairly well with the weather coefficient at the time the readings on models *F* 2, 12 and 3 were made.

The observations on this set of models extended over a considerable period of time, one to four complete sets of observations being made each day. In all, a total of about forty sets were made. It will be impossible to give in detail the data obtained, hence, summarizations will be presented from which the necessary conclusions may be drawn.

It was pointed out in connection with the data presented in Tables I. and II. that the values of O_r, the optimum reflection factor, agreed fairly well with the weather coefficient. Theory indicates that this should occur. The data will first be summarized in such a way as to show the experimental verification of this point.

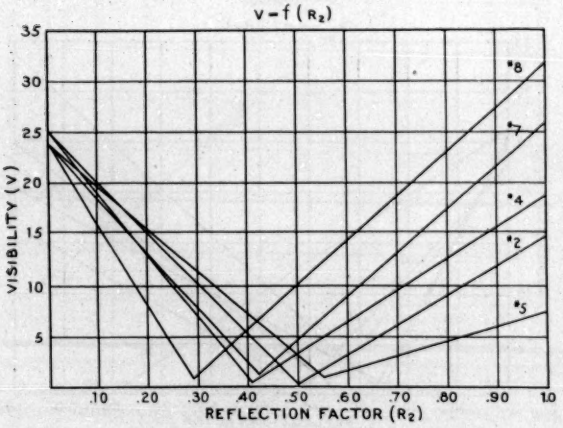
In Table III. are presented the results of ten sets of data on the nine black to white models, *F* 2 to *F* 6. Immediately under each model number is placed the reflection factor value for that model. In column *W* are the values of the weather coefficient for each set and in column O_r, the values of the optimum reflection factor as determined by interpolation from the plotted curves.

TABLE III.

Set No.	*F* 2	*F* 12	*F* 3	*F* 13	*F* 4	*F* 14	*F* 5	*F* 15	*F* 6	*W*	O_r
	.75	.60	.45	.36	.28	.20	.15	.10	.05	....	...
1	15.6+	10.1+	4.5+	1.0+	5.8−	9.4−	13.−	17.−	20.−	.38	.36
2	7.4+	3.0+	4.0−	7.7−	11.0−	15.2−	18.−	19.−	22.−	.47	.50
3	21.8+	16.0+	10.0+	5.0+	1.3+	6.2−	10.−	14.−	22.−	.30	.27
4	11.0+	5.0+	1.0+	5.0−	7.9−	12.0−	16.−	18.−	23.−	.40	.41
5	3.7+	1.5+	5.0−	7.5−	12.0−	15.0−	17.−	19.−	22.−	.59	.55
6	10.0+	6.0+	1.7+	4.2−	7.0−	10.9−	13.−	16.−	22.−	.42	.41
7	16.1+	7.7+	1.2+	5.0−	10.0−	12.0−	16.−	20.−	23.−	.40	.42
8	22.0+	12.5+	8.5+	3.0+	2.5−	7.0−	12.−	15.−	20.−	.30	.29
9	12.0−	6.3−	12.0−	14.2−	18.0−	19.0−	21.−	24.−	25.−	.77	.80
10	13.6+	10.0+	6.4+	4.0+	1.5+	5.5−	10.−	15.−	22.−	.30	.25
ean	12.3+	7.8+	5.4+	5.7−	7.7−	10.2−	14.6−	17.7−	22.−	.433	.426

In Figs. 9 and 10 are plotted the data from Table III., the number attached to each curve being that of the set of data used in obtaining it. A careful consideration of these curves reveals several interesting facts. It will be noticed that in general the value of V at which the positive and negative elements of a given curve intersect is not zero. This means that there is no value of R_2 which results in complete invisibility. This is due to the fact that a color contrast existed between the models and the sky

FIG. 9.



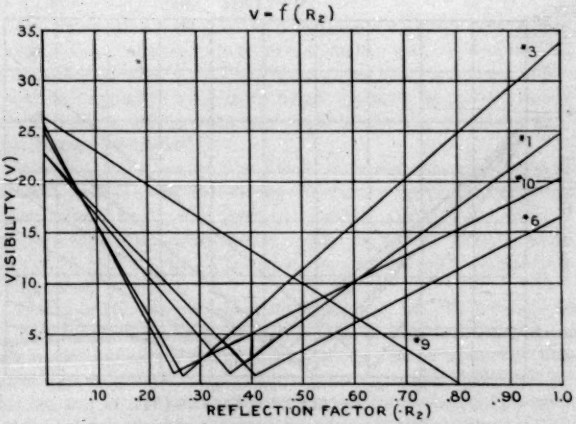
Variation of visibility with reflection factor for tests 2, 4, 5, 7, 8.

background, resulting in a residual visibility. Therefore, a pure gray cannot be expected to give the minimum visibility.

The curves plotted in these Figs. 9 and 10 are of the same type as the theoretical curves representing V as a function or R_2. Now, in computing the values from which theoretical curves representing $V = f(R_2)$ were plotted a value of $k = 1.02$ was taken. This gave for the point from which the elements of negative slope diverge, the point, $Y = 0$, $X = +50$, and $Y = 0$, $X = -50$ as the origin of the elements of positive slope. Now in Figs. 9 and 10 it will be noted that the elements of negative slope converge

to a common point, $Y = 0$, $X = +25$. By using this value in Equation 6 and solving for k, a value of $k = 1.04$ is obtained. This indicates the value of $k = 1.02$ was too low for the conditions that exist in the operation of the visibility-meter. In order to compare experimental results with the theoretical requirements, this indicates that a value of $k = 1.04$ should be used in computing the theoretical curves. It will be noted that the agreement between W and O_r is in general quite good and the agreement between the means of the ten sets (Table III.) very good.

FIG. 10.

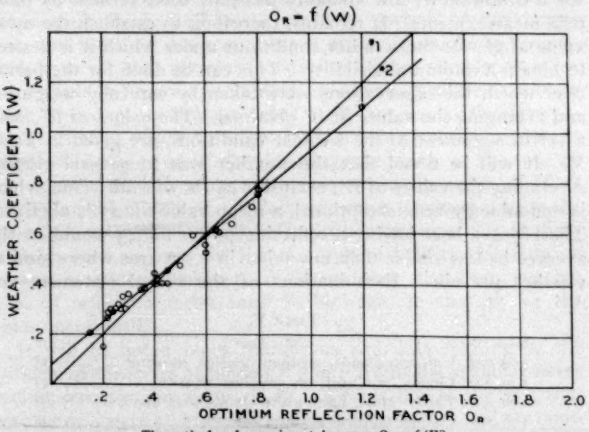


Variation of visibility with reflection factor for tests 1, 3, 6, 9, 10.

Additional data on the agreement between W and O_r are given in Table IV. The values of O_r were obtained in the same way as those in Table III., *i.e.*, by interpolation from curves, $V = f(R_2)$, plotted as illustrated in Figs. 9 and 10. Examination of the data in Tables III. and IV. shows that for low values of W, O_r is usually too low, while for high values of W, O_r is, in general, too high. The agreement between the two for intermediate values ($W = .40$ to $.60$) is quite good. In order to show this relation in graphic form the curves in Fig. 11 were plotted. Theoretically the curve, $W = f(O_r)$, should be a straight line at 45°

to the axis of reference, Curve 1. The positions of the points plotted from the data in Tables III. and IV. are indicated by the small circles. The most probable position of the experimental curve ($W = f(O_r)$), is the line 2. It will be noted that the experimental curve is of lesser slope than is indicated by theory, but that

FIG. 11.



Theoretical and experimental curves, $Or = f(W)$

it crosses the theoretical curve at the point $W = .51$. This discrepancy between the theoretical and experimental curves has not as yet been accounted for. However, it is relatively small, especially within intermediate values, $W = .35$ to $W = .65$, within

TABLE IV.

W	O_r	W	O_r	W	O_r
.20	.15	.38	.35	.52	.60
.26	.22	.40	.45	.55	.60
.29	.22	.40	.43	.60	.65
.29	.24	.40	.45	.61	.64
.30	.23	.40	.45	.63	.60
.32	.26	.44	.40	.70	.78
.36	.30	.45	.45	.75	.80*
.35	.28	.45	.45	1.10	1.20*

which region the great majority of the measured values of W are found to lie. The data derived from Tables III. and IV. are not suitable for deciding upon the mean value of W, since these were chosen simply to cover the entire range of the observed values of W.

The close agreement between W and R_2 called for by theory for a condition of low visibility has now been verified by practical measurements. It remains, therefore, to establish the mean value of W for the weather conditions under which it is desired to obtain a minimum visibility. This can be done for the period over which the observations were taken by carefully weighting and averaging the values of W obtained. The values of W, with a verbal statement of the weather conditions, are given in Table V. It will be noted that the weather was in general cloudy. Averaging the values of W, exclusive of the seventh value, which is undoubtedly very exceptional, a mean value of .43 is obtained. Therefore, a boat having a reflecting power of .43 would on the average be less visible than any other in a sea area where similar weather prevails. Examinations of the verbal statements of

TABLE V.

Hour	Weather	W
3.00 P.M.	Uniform clouds, medium density, showing	.38
3.30 P.M.	Cloudy, not uniform, medium density	.45
10.00 A.M.	Fairly clear, light white clouds over sun	.31
4.00 P.M.	Cloudy, uniform medium density	.48
3.00 P.M.	Heavy dark uniform clouds all over	.59
4.00 P.M.	Sky covered with dark clouds	.46
	Light streak above the horizon	2.70
4.30 P.M.	Dense uniform clouds all over	.80
2.30 P.M.	Clear, light white clouds, local	.30
10.00 A.M.	Clear sky, light gray fog bank over lake	.36
2.30 P.M.	Sky perfectly clear, no fog	.30
10.30 A.M.	Clear light gray fog over lake	.26
3.30 P.M.	Clear, no clouds	.22
4.00 P.M.	Cloudy, uniform gray clouds	.45
9.30 A.M.	Partially clear, foggy	.50
4.00 P.M.	Light clouds, sun obscured	.48
9.30 A.M.	Partially cloudy, no sun	.50
2.00 P.M.	Light veiling clouds over sun	.35
3.00 P.M.	Light veiling clouds over sun	.45
4.00 P.M.	Light veiling clouds over sun	.53
	Mean	.43

weather show that only five of the twenty observations were taken under clear conditions, the remaining fifteen being for cloudy or partly cloudy weather. That is, 75 per cent. of the observations apply to cloudy or partly cloudy conditions.

Now a careful analysis of weather reports covering the region in which a majority of the sinkings have been occurring shows that 70 per cent. of the days are cloudy. Hence the factor .43 should apply fairly well to the northern portion of the danger zone, and south of it where there is mist. It should be emphasized, however, that in order to establish a more reliable value for O_r, it would be necessary to obtain precise and definite data from which to deduce an average value of W for any particular submarine-infested region through which the ship to be painted must travel. This value of W should, of course, apply to the time of year considered. Even on clear days, when the value of W is low during the middle of the day, this value increases during the hours just after sunrise and before sundown, especially if there be some haze or mist in the atmosphere. With the data at present available, however, it seems that the best average value of reflection factor must be between .40 and .45 for the region mentioned.

It will be noted from the curves and data relating to the visibility of the series of models F 2 to F 6 that the visibility never reaches zero for any value of W. This is due to the fact that these models were all gray, the saturation being zero and the dominant hue indeterminate; thus color difference always existed between the object and background, giving rise to a residual visibility due to color contrast.

The next step in the work was the determination of the best hue and saturation to give low visibility. The most logical and scientific procedure for the determination of these factors would have been to measure by means of a colorimeter the hue and saturation of the sky. If such measurements could have been made under all conditions of weather and over a considerable period of time, and the results averaged for the desired period and weather conditions, it would have been possible to make up the desired color directly. However, it was thought best and most expedient to obtain the desired information by a method of trial and error. It was evident from the direct visual observations made over a considerable period that the average value of

the dominant hue was situated in the blue-green region and that the saturation was on the average quite low. Several models were therefore made up by painting with various hues of blue-green and of low saturation. They were mounted on the rack in the usual way and observed. One model in the group first prepared was found to be of approximately the correct hue, but the saturation was obviously too high.. A second set of models was then prepared, using the best hue as already determined, but of various saturations. In this way, by a series of consecutive approximations, a hue and saturation were obtained which by observation over a considerable period and under various conditions of the weather was considered satisfactory. The reflection of all models used in this work was kept at the best value of that quantity as previously determined, namely, .40 to .45. The color of the model *F* 20, which was proven to be the best by observation, was then measured on the colorimeter and found to be represented by the specification: Hue (wave length of dominant hue) = 488$\mu\mu$; Saturation (per cent. white) = 88 per cent.

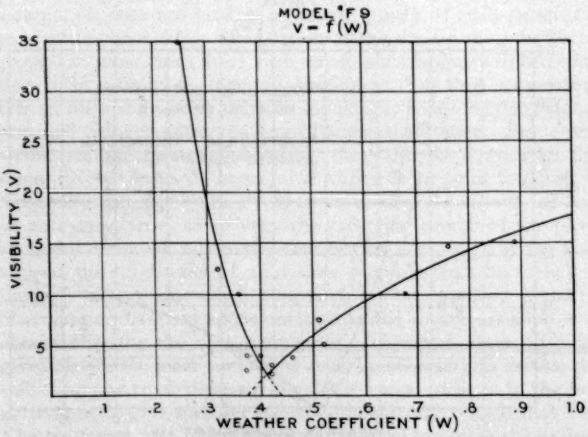
TABLE VI.

F 9		*F* 20		R_1 = .43	
W	*V*	*W*	*V*	*W*	*V*
.37	4.0+	.40	1.5+	.10	81.5+
.52	5.0−	.47	2.5−	.20	27.8+
.46	3.8−	.70	10.0−	.25	17.0+
.51	7.5−	.90	11.5−	.30	9.9+
.68	10.0−	.30	12.5+	.35	4.6+
.48	1.0−	.34	5.0+	.40	0.9+
.87	15.0−	.25	2.5+	.414	0.0
.76	14.0−	.42	2.5+	+.0	0.0
.30	20.0+	.38	4.0+	.447	0.0
.36	10.0+	.55	5.5−	.50	2.6−
.40	4.0+	.42	0.9+	.60	6.3−
.25	35.0+	.45	1.0−	.70	9.0−
.40	1.0+	.78	9.0−	.80	11.0−
.42	3.0−	.35	8.5+	.90	12.6−
.32	12.5+	.50	3.0−	1.00	13.8−

The data on Model *F* 9 are given in Table VI., column *F* 9, these being the visibility values determined under various weather conditions. In this case the model is constant, while *W* varies. Hence, the data plotted as a curve must be the form of $V=f(W)$, as shown in Fig. 12. The minimum visibility is 2.2 at R_2=.41. The

residual visibility in this case was due to saturation contrast. The data on model *F* 20 are given in column *F* 20, Table VI. and the curve plotted from them in Fig. 13, Curve 1. It will be noted that in this case the minimum visibility is very low (0.3) and that this occurs at $R_2 = .43$. In order to determine how nearly the experimental curve, $V = f(W)$, for this model agrees with the theoretical, the curve shown as a dotted line and numbered 2 in Fig. 12 was plotted. The fact that the minimum visibility of model *F* 20

FIG. 12.

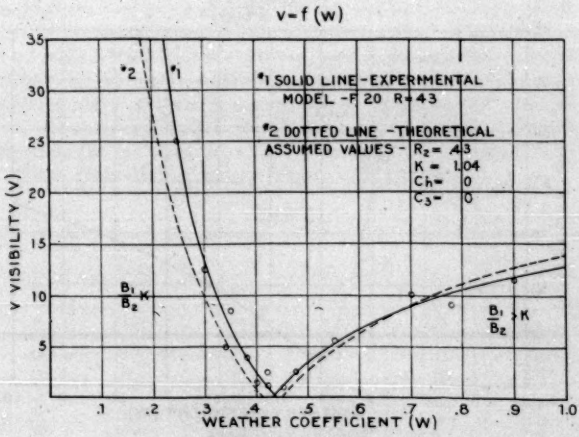


Visibility test of Model F9.

as shown by Curve 1, Fig. 13, was practically zero shows that the hue contrast, C_h, and the saturation contrast, C_s, between model and background must have been very small. Hence, the curve, $V = f(W)$, for the model should agree quite closely with the theoretical curve, $V = f(W)$, for brightness contrast alone. Taking from previous experimental results that $R_2 = .43$ (the reflection factor of *F* 20), solution of Equation 12 for various values of W gives the data, Table VI., column $R_2 = .43$, from which Curve 2 was plotted. It will be noted that the two curves agree quite well for the region when $W = .45$ to .90, but that some

difference occurs for lower values of W. This is probably due to the presence of a color contrast which existed only under those conditions of weather resulting in low values of W. It is very evident that there will be weather conditions for which the color of this model, F 20, will not be correct either in hue or saturation, but from the data available it seems quite certain that this specified color will be best, from the standpoint of low visibility, for any locality where a large proportion of the days are cloudy or partially cloudy.

FIG. 13.



Visibility test of Model F20.

It should be borne in mind that the visibility specifications expressed above are based on the following hypotheses:

(1) Background composed entirely of sky immediately adjacent to horizon line.

(2) Predominating weather condition entirely or partially cloudy, $W = .40$ to $.45$.

(3) Visible surfaces of object uniformly illuminated by natural light, that is, sunlight more or less diffused by clouds.

(4) If the surfaces are such that the value of R depends upon the conditions of illumination and the direction of observa-

tion, the value of R referred to in the specifications above is that particular value obtained by measurement under the existing conditions of illumination and from the direction of the specified point of observation.

The first hypothesis represents the condition existing in practically every case of the observation of a surface craft from a submarine. The second conforms with the conditions in a large and most hazardous part of the present submarine danger-zone. This conclusion is based upon a careful summary of all the available data relating to the weather of these regions. The third assumption does not hold at all for an object of such structural peculiarities as a ship. The portion of the sky forming the background in most cases is of such small, solid angular dimensions that it is usually uniform in brightness, hue and saturation. Hence, for low visibility the boat should also be uniform in brightness and color as observed from the specified point. For every visible surface element, therefore, the value of B must be the same. Now, $B = E \cdot R$, hence, if E varies from point to point, R must vary also, and inversely, in order to keep B constant. It will be necessary, then, to so vary the reflecting power of the paint applied to the areas where the illumination is too high or too low, so that the brightness of all visible surfaces shall be as nearly uniform as possible when viewed from the specified point of observation. A good approximation to the fourth assumption is obtained by the use of a matt paint. However, since no paint gives absolutely matt surface, it may be necessary to reduce the reflecting power of these surfaces that are so inclined to the line of sight as to give specular reflection in the direction of that line. The treatment of a given structure required to obtain the desired result necessitates the careful study of the individual structure and of the position, shape and relative intensity of the high lights and shadows under the lighting conditions for which it is desired to obtain low visibility. This treatment should be such that the projected effect, that is, the apparent brightness, hue and saturation, as measured from the specified point of observation, shall be equivalent to that of a perpendicular surface (exposed to the full illumination of the natural existing conditions) uniform in reflection factor, hue and saturation.

In order to determine the relative merits of the camouflage schemes proposed by various individuals and firms for the pro-

tection of surface craft from attacks by submarines, a large number of visibility measurements were made. The models used were of the standard profile type, and were prepared by the various parties advocating the use of these systems of protective coloration.

The principles upon which these various systems are based vary greatly from group to group, but for the purposes of this discussion they will be divided into two general classes: (1) those in which the size of the individual unit employed in carrying out the color scheme is below the resolving power of the eye when at a distance of 6000 yards, (2) those in which the size of unit is above the resolving power of the eye at a distance of 6000 yards. In case of the first class, no matter what the shape or color of the units used, the model as observed from Station 1 appeared of a uniform brightness, color and saturation when the pattern used was regular. In case the pattern was irregular, certain variation in general tone could be detected, but no units were visible as such. This system of coloration is based upon the assumption that a surface so painted will be less visible than a surface painted with a solid color of the same reflecting power, hue and saturation as results from the blending, due to loss of resolution, of the units in a broken color system. From the physical standpoint there seems to be no support for such an assumption; but it was considered advisable to make a large number of measurements in order to prove or disprove its validity. This system is sometimes referred to as "vibration," or "scintillation" system, although the exact sense of these words used in this connection is not entirely clear. The models designated by the letters *A, D, E* and *H* fall into this classification, although it should be remembered that such classification is not at all rigid.

A large number of readings were made on the models of these various groups, but it does not seem desirable to present all of these data at this time. However, in order to compare directly the merits of these different systems of coloration, a summary of data is made in Table VII. The data are tabulated in columns designated as follows:

Model No. = The identification number.
W_m = The value of weather coefficient for which visibility is minimum.
V_m = Minimum visibility.
$V_{.43}$ = Visibility of the model when $W = .43$.

V_x = Average visibility for the range $W = W_m + .10$ to $W = W_m - .10$.

Color = Verbal indication of whether the model was painted with a solid or broken color scheme.

Unit = Designation of the size of unit employed in case a broken color system was used. "Small" indicates that the units were below the resolving power of the eye when the model was at a distance equivalent to 6000 yards. "Large" indicates that the units were above the resolving power of the eye at that distance. In the case of Model G1 the units were large, but due to the very low contrast between units they were not visible at the specified distance. Hence this model to all intents and purposes should be regarded as belonging either to the solid color class or to the small unit, broken color group.

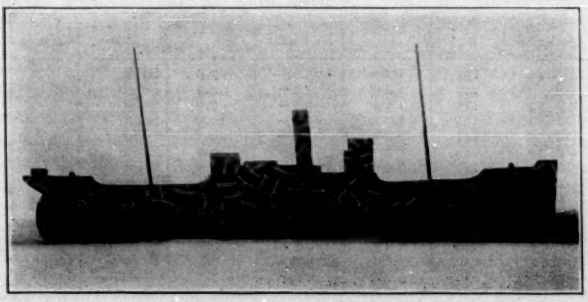
TABLE VII.

Model No.	W_m	V_m	V_{43}	V_x	Color	Units
F 12	.51	1.2 ±	5.6 ±	3.0	Solid	
F 3	.42	1.4 ±	2.6 −	4.3	Solid	
F 13	.35	1.6 ±	5.5 −	7.0	Solid	
F 4	.31	1.3 ±	9.0 −	10.5	Solid	
F 9	.41	2.2 ±	3.0 −	6.1	Solid	
F 20	.43	0.3 ±	0.3 ±	3.2	Solid	
R 43	.43	0.0 ±	0.0	2.0	Solid	
A 5	.38	1.5 ±	4.7 −	5.3	Broken	Small
B 3	.31	3.0 ±	7.0 −	7.8	Broken	Large
C 1	.32	3.0 ±	7.5 −	8.6	Solid	
D 1	.49	2.0 ±	7.2 +	5.2	Broken	Small
E 1	.49	4.2 ±	7.5 +	6.6	Broken	Small
G 1	.30	1.0 ±	10.0 −	7.8	Broken	Large
H 1	.30	3.0 ±	9 5 −	10.5	Broken	Small

Now, in discussing the data given in Table VII., it is desired to emphasize the significance of the term V_x. It was suggested in support of the small unit system of broken color that although such a method of painting might not produce a lower minimum value of visibility, possibly the average visibility for a fairly large variation in the weather coefficient might be less than in case of a solid color. This possibility is based upon the assumption that the mean reflecting power, hue and saturation of a surface covered with a series of small colored patches will, when subject to a given variation in the intensity and quality of the incident light, change according to laws other than those governing the change in reflection factor, hue and saturation of a solid color when subject to the same variation in the illumination. In order to test the validity of such an assumption, the value of V_x was computed

for the various models. Since the position of minimum visibility is not the same for all models, no fixed range of W values could be used for a comparison upon this point. It was necessary, therefore, to consider a given range of W values above and below W_m for each individual model. The data obtained indicate that the broken color systems show no advantage over the solid colors. A considerable variation in the V_s values does occur, but it will be noted that this is a function of W_m and not of the solid or broken characteristics of the color scheme. A consideration of the theoretical curves shows that V_s should vary according to some inverse function of R_2.

FIG. 14.

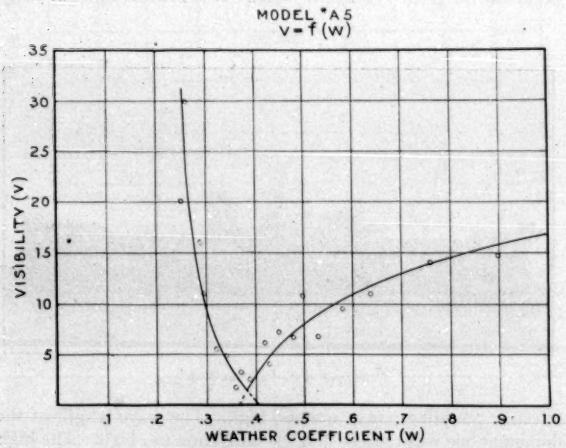


Photographs of a Herzog Model.

In order to test the relative merits of the broken and solid color systems, models of exactly the same mean reflecting power, hue and saturation, should be compared. No models precisely filling these requirements were found among those used, numbers F 20 and A 5 being those that most nearly approach these conditions. It will be noted that F 20, solid color, shows a lower average visibility than A 5. The difference between the two values is not great and is due partially to the fact that W_m is higher for F 20, and partially to the lower color contrast between model and background in case of F 20. The conclusions to be drawn are, therefore, that visibility is a function of mean reflecting power, hue and saturation, and that nothing is either gained or lost by the use of a system of broken colors.

While it is not considered necessary to give in detail all of the data on which the above conclusions were based it may be of interest to present some of this data, together with the curves plotted therefrom, and photographs of the models upon which the observations were made. This will serve to give the reader some idea of the type of models examined and the shape of the curves obtained by plotting visibility as a function of "weather coefficient." In Table VII. are given the data on models *A* 5, *D* 1, *E* 1, and *H* 1.

FIG. 15.



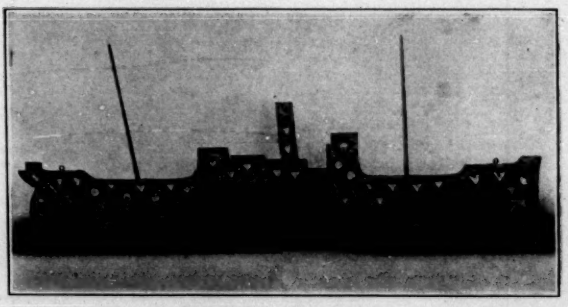
Visibility test of a Herzog Model.

The data on the model *A* 5 (Herzog system) shown in Fig. 14, are given in column *A* 5, Table VII., and the curve, $V = f(W)$, is plotted in Fig. 15. It will be noted that the minimum value of visibility occurs at the point, $V = 1.5$, $W = .38$. The mean effective reflection factor is, therefore, equal to .38, which is too low for the average value of W (.43) previously determined. The curve is of the same general type as those obtained from the models of solid color (the *F* group), and shows no superiority in any respect. The hue and saturation of this group of models compared quite satisfactorily with the last model of the *F* group,

F 20. The dominant hue was, in general, too far toward the red end of the spectrum, although this was not true for all models of the group.

Model D 1 (Pleuthner) is shown in Fig. 16. The pattern was rather irregular and blended completely at 6000 yards into a uniform brightness, hue and saturation. The resultant hue was yellow and of fairly high saturation. The reflection factor was also high. The data are given in Column D 1, Table VII., and the resulting $V = f(W)$ curve in Fig. 17. The minimum visibility occurs at the point $V = 2.0$, $W = .49$. It is evident that the mean

FIG. 16.

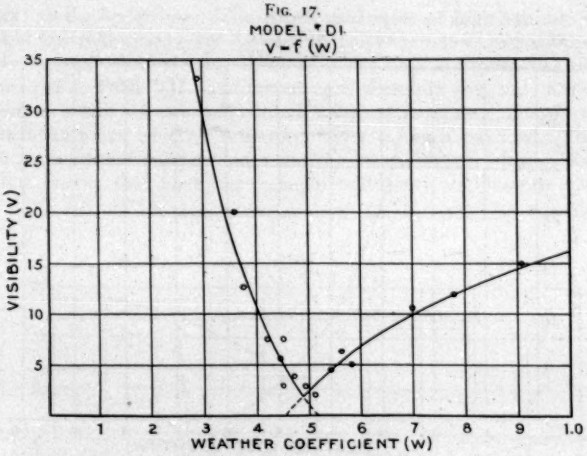


Photograph of the Pleuthner Model.

effective reflecting power was too high. The wave length of the dominant hue was too low and the saturation too high. The high value of minimum visibility is due to a combination of hue and saturation contrast.

Model E 1 (Gomez) is shown in Fig. 18. This model was painted with horizontal stripings, blending gradually one into the other, the average reflecting power increasing from the water line upward. The intention evidently was to imitate the horizontal stratification of clouds sometimes observed just above the horizon. The angular dimensions of these stripes were entirely too small at a distance of 6000 yards to simulate any natural stratification ever observed by the writer. In fact, at the distance mentioned, the striping was not visible at all, the blended effect being of a

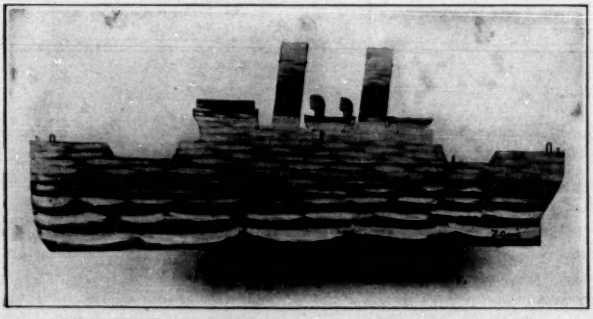
FIG. 17.



Visibility test of the Pleuthner Model.

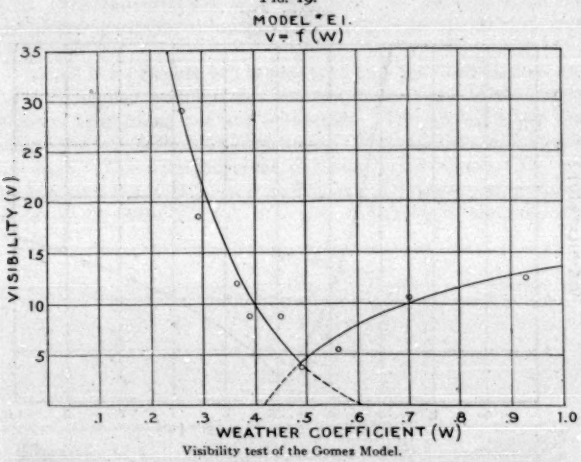
gradual diminution in brightness upward from the water line. The data are given in Column E 1, Table VII., and the curve, $V = f(W)$, in Fig. 19. The minimum value of visibility occurs at

FIG. 18.



Photograph of the Gomez Model.

FIG. 19.

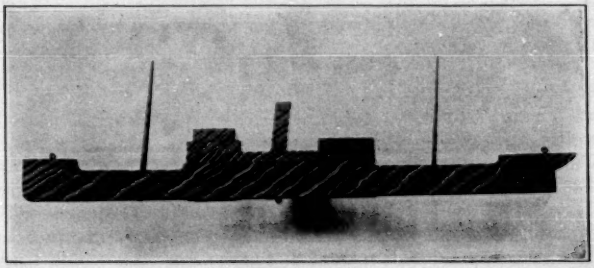


Visibility test of the Gomez Model.

$V = 4.5$, $W = .49$. The mean effective reflection factor was too high, the saturation satisfactory and the wave length of the dominant hue too low, the mean hue being yellowish.

Model *H* 1 (Patterson-Sargent Company), shown in Fig. 20, was painted with wavy diagonal stripes inclined at an angle of

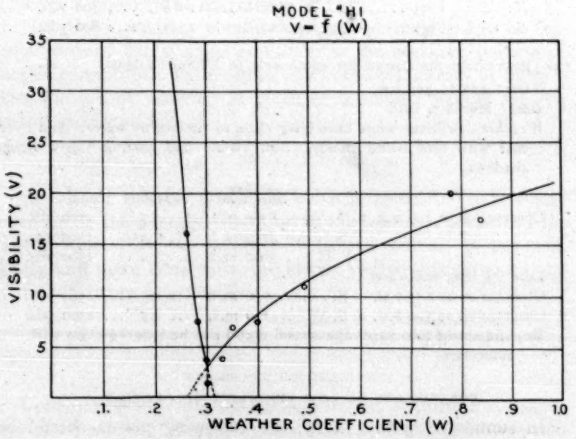
FIG. 20.



Photographs of the Patterson-Sargent Company Model.

45° to the water line. The colors used were of high saturation and low reflecting power. These stripes were not visible at 6000 yards, equivalent viewing distance. The data are given in Column *H* 1, Table VII., and the curve is shown in Fig. 21. Minimum visibility occurs at $V = 3.2$, $W = .30$. It is evident, therefore, that the mean effective reflection factor is much too low. The hue was rather indeterminate and the saturation very low. The few curves that have been shown will serve to illustrate the

FIG. 21.



Visibility test of the Patterson-Sargent Company Model.

type of data obtained, the method of interpretation and the conclusions drawn from an examination of the visibility-weather coefficient relation.

The United States Navy Department placed at the disposal of the Submarine Defense Association the U. S. S. *Gem* for its investigations. The vessel was painted Omega gray on one side, the color which our experiments indicated was the best for low visibility in the northern portion of the danger zone. To confirm these conclusions, several observations of the U. S. S. *Gem* for

visibility were taken at sea off New London. The following are two records:

(1) Observation Station on the deck of a submarine. (Long Island Sound off New London.)

Time: 10.30 A.M.
Date: March 10, 1918.
Weather: Very foggy.

Direction	Side	B (Sky Brightness)	Visibility	Distance
N	Starboard	1450 m. l.	2.8	500 yds.
S	Starboard	1450 m. l.	.1	800 yds.
N	Port	1450 m. l.	1.8	500 yds.
S	Port	1450 m. l.	.1	800 yds.

(2) Observation Station on the south side of Fisher's Island.

Time: 2.15 to 3.20 P.M.
Date: March 5, 1918.
Weather: Brilliant white haze lying close to surface of water. Sun overcast with thin white clouds. Sun visible and casting fairly strong shadows.

	B (Sky Brightness)	Visibility	Distance
Observation at 2.15 P.M. to S. W.	6280 m. l.	1.5	1500 yds.
S.	4750 m. l.	.45	1500 yds.
S. E.	3600 m. l.	.2	1500 yds.
Less fog over the water:			
Observation at 2.53 P.M. to S.	3750 m. l.	2.5	1500 yds.
Observation at 3.20 P.M. to S. W.	4120 m. l.	4.3	1500 yds.

Beyond about 3000 yards the vessel could not be located even with binoculars.

CONCLUSIONS ON LOW VISIBILITY COLORATION.

In summarizing this study, the following points should be emphasized. The experimental observations show that:

(*a*) Broken color systems employing contrasting units so large as to be visible as units at the distance considered are objectionable. The contrast between juxtaposed units or between some of the units and the sky background tends to increase visibility.

(*b*) Broken color systems employing units so small as to be invisible as such at the distances considered are neither advantageous or detrimental. The visibility in such cases depends entirely upon the mean effective reflection factor, hue and saturation of the surface, resulting from the reflection factor, hue

and saturation of the surface, resulting from the blending of the units of the design into an apparent uniformity at the distances considered.

(*c*) A solid color system produces low visibility when

$$R_2 = W,$$
and $$S_2 = S_1,$$
$$H_2 = H_1.$$

(*d*) The average visibility, V_x, for a given variation in W, varies according to some inverse function of R_2; the greater the value of R, the less the magnitude of V_x.

(*e*) For a weather condition which is considered to be representative of the average condition in the northern portion of danger zone, where the weather is 70 per cent. cloudy or there is mist, the lowest visibility is given by a surface having the following specifications:

	Hue	Saturation	Reflection Factor
Omega (ω) Gray....	488 $\mu\mu$	88 per cent.	$W = .43$

(*f*) For a weather condition which is considered to be representative of the average condition in the southern portion of the danger zone when the weather is 40 per cent. to 50 per cent. cloudy and there is no mist, the lowest visibility is given a paint by the following specifications:

	Hue	Saturation	Reflection Factor
Psi (ψ) Gray....	478 $\mu\mu$	88 per cent.	$W = .35$

DECEPTIVE COLORATION.

Considering now the second subdivision of protective coloration as a means of defense against submarines, it is evident that a problem of quite different type confronts the investigator. This problem appears to be less amenable to exact scientific treatment than that of low visibility, at least being less physical and more psychological in nature. It is quite evident that, if any deceptive effect is to be produced the elements of the design employed must present relatively great contrasts either in brightness or color and must be of sufficient size as to be visible at the distances at which it is desired the deception shall become operative. If such contrast exists, some of the elements of the pattern must present a marked contrast with the sky background as well as with the contiguous units of the design. This being the case, such elements must be

distinctly visible as such, thus betraying the presence of some object not naturally a part of the sea or sky. No matter how grotesque the shape or color of such elements, it seems almost certain that any intelligent submarine commander observing them and knowing that ships are being protectively colored would interpret them as indicating the presence of a ship of some kind. It is, therefore, certain that such coloration is directly antagonistic to the production of low visibility, thus decreasing the probabilities of a ship escaping detection and increasing the chances of an attack being made.

It does not appear that anything is to be gained by painting a ship to look like any object other than a ship or so as to make it appear merely as a grotesque shape. Any plan of deceptive coloration to be effective must operate in such a way as to produce some definite effect such as to confuse or mislead the submarine commander in the measurements or estimates upon which his subsequent actions in making the attack are based. Thus, any deception regarding the speed, course or range of the craft to be attacked resulting from the scheme of deceptive coloration employed will tend to increase the probabilities of the failure of the attack.

A large number of optical illusions have been collected from the psychological literature and examined from the standpoint of applicability to the production of deception under the conditions existing in this problem. Having considered the methods of observation used by the submarine commander and the nature of the apparatus at his disposal for obtaining these observations, it seems that the only type of illusions offering any possibility of advantageous use in connection with this problem are those based upon the principle that a deception regarding the course of the vessel so painted might be produced.

Since deceptive coloration requires the visibility of pattern to accomplish its object, and low visibility coloration requires uniformity in order to match the sky background, it is evident that in general the two methods are directly antagonistic. In spite of this, it is possible to reconcile the two systems and to devise a system of protective coloration based upon the principle of both "low visibility" and "deceptive" coloration. This possibility is due to the fact that the eye fails to resolve the images of objects when such objects subtend very small visual angles. It is due to

this fact that a surface covered with a pattern made up of units contrasting either in brightness or color or both appears as a surface uniform in brightness and color when viewed at a sufficiently great distance. The resolving power of the eye or of any optical instrument depends upon the aperture ratio of the system. Assuming a pupillary diameter of 4.0 mm., the resolving power of the eye expressed in terms of visual angle is approximately 30" of arc. This, in terms of linear dimensions, corresponds to a distance of .15 mm. at one metre or 10 inches (24 cm.) at one mile ("Outlines of Applied Optics," P. G. Nutting, p. 65). These values apply to conditions of very great contrast between object and background and in case contrast is low the visual angle for resolution is probably somewhat larger.

By taking advantage of this failure of the eye to clearly resolve objects of small visual angle, a design may be constructed consisting of contrasting elements which will be invisble at any desired distance. The distance at which these elements of design are just below the resolving power of the eye may be termed the "blending" distance. A surface on which such a design exists will, at distances greater than the blending distance, appear to be uniform in brightness and color, while at distances less than that the pattern will be clearly visible. Now, by properly adjusting the brightness, color and relative sizes of the units of any such design, the surface may be made to appear, when viewed from beyond the blending distance, as a uniformly colored surface of the proper brightness and color to give minimum visibility, while at closer range the elements of the design will be visible and may be so arranged as to produce a deceptive effect.

Two colors have been worked out which, when applied to equal areas of any surface, will, when viewed at distances greater than the blending distance of the pattern, result in the low visibility color, that is, the Omega (ω) gray. The two components of this pair of colors are given by the following specifications:

	Name	Hue	Saturation	Reflection Factor
(1)	Blue	474$\mu\mu$	95 per cent. white	.59
(2)	White	554$\mu\mu$	78 per cent. white	.26

It should be emphasized that these two colors must be applied to equal areas of the surface considered if it is to produce the ω gray beyond the blending distance, at which the transition from the deceptive design to low visibility color occurs.

In order to produce the Psi (ψ) gray, which gives low visibility in regions where the average value of W is .35, another pair of colors was established, being specified as follows:

Name	Hue	Saturation	Reflection Factor
(1) Blue	476$\mu\mu$	51 per cent. white	.11
(2) White	484$\mu\mu$	95 per cent. white	.56

These colors also must be applied to equal areas of the surface considered if it is desired to produce the ψ gray at or beyond the blending distance. The size of the units of the design must be adjusted to the proper size in order to obtain the desired value of blending distance. It is the writer's opinion that the most effective type of protective coloration will eventually be found to embody some such combination of the two general types.

In closing this paper no attempt to present a complete summary will be made, although it may be well to recall a few of the most important points. The subject of protective coloration quite naturally divides itself into two main divisions: "low visibility" coloration and "deceptive" coloration. These in general are antagonistic in principle.

The subject of low visibility has been quite thoroughly treated both from the theoretical and practical points of view. An instrument for the measurement and numerical specification of the visibility of an object under any condition of lighting has been devised and constructed. A large number of measurements of the visibility of profile models under conditions of natural lighting have been made. As a result of this work, two colors designed to give minimum visibility in different parts of the danger zone have been developed and specified. For that part of the zone where 70 per cent. of the days are cloudy or partially so, the color designated as ω (Omega) gray will give minimum average visibility, while for that region where 40 per cent. of the days are cloudy or partially so the color designated as ψ (Psi) gray will give the best results. In order to obtain the lowest possible visibility the boat should be so painted that from the desired point of observation the projected effect is equivalent to flat surface conforming to the above specifications. This necessitates certain modifications of structural details as well as the modification in the constitution of the paint applied to various parts of the structure in order to eliminate regions of shadow and high light.

The possibility of combining the two general systems, of protective coloration, low visibility and deceptive coloration, is pointed out and two pairs of colors suitable for such use are specified. In general, it is concluded that the probability of a filter being found to obviate the effect of the protective scheme is much greater in case of deceptive patterns composed of units exhibiting high saturation and hue contrast than in the case of low visibility coloration or of deceptive designs employing units of low saturation contrast.

In closing, the author wishes to acknowledge his indebtedness to Mr. Lindon W. Bates of the Submarine Defense Association (under whose auspices this work was carried out) for his coöperation and encouragement given so freely throughout the course of the work, and also to Dr. C. E. K. Mees, Director of the Research Laboratory of the Eastman Kodak Company, for his many helpful suggestions, and to Mr. Prentice Reeves, also of this Laboratory, for his able assistance in carrying out the experimental part of the work.

ROCHESTER, N. Y.
May 1, 1919.

The Magnesite Industry in the United States.—Magnesite (Magnesium Carbonate) is largely employed in the steel industries, and the United States is the largest consumer, taking about 50 per cent. of the world's output. Before the war fully 90 per cent. of this was imported, the bulk coming from Austria-Hungary, and the balance from the Grecian quarries. With the outbreak of the war the Austrian supplies were at once cut off, and after 1916 those from Greece were seriously curtailed. In 1917, the domestic consumption was over 355,000 tons valued at more than $3,700,000, and nearly 90 per cent. of this was obtained within the United States. A great new industry was developed in the State of Washington, while the California industry was very much expanded. A magnesite industry sprang up in Quebec, and although the product was inferior in quality to that produced in the United States, yet the nearness to the great eastern manufacturers made this source competitive. The chief handicap to the western magnesite industries is the long railroad haul, competing with cheaper ocean carriage. The American magnesite is purer than the Austrian, the latter having a notable iron content, but this impurity is somewhat of an advantage in the steel industries. This objection, however, has been satisfactorily overcome by adding a small amount of an iron compound to the American product.

The restoration of shipping conditions will bring about a formidable competition with the American products, and while some of the magnesite from Venezuela may compete with the European output, yet the latter, especially the Austrian, will dominate the market if anything like pre-war prices prevail. Under these conditions the western output would be limited to the market west of the Mississippi. The precise line of contact will depend on several conditions, but the concentration of the steel industries regions east of the Mississippi will give great advantage to the foreign producer.

The question, therefore, becomes of considerable difficulty when the tariff problem is brought in, and data upon the subject have been collected by the United States Tariff Commission, and printed as a bulletin of information, from which publication the above data are taken. H. L.

Mineral Production of the United States in 1918. (*U. S. Geological Survey Press Bulletin No. 422.*)—The total value of the minerals produced was about $5,526,000,000, more than half a billion dollars in excess of the value recorded for 1917, but the total quantity produced was less. The output of fuels was greater than in 1917, though somewhat less anthracite coal was marketed. The increase in the quantity of coal marketed was about 5 per cent., but the increase in value, due to higher prices, was more than 17 per cent. It is significant that though the increase in the quantity of petroleum marketed was only a little more than 4 per cent. the increase in value was over 32 per cent.

The value of the metals produced was about 3 per cent. greater in 1918 than in 1917. The figures show that less iron ore and steel were produced, but here again values were higher. A little more pig iron was made, though the quantity shipped was less. Copper and zinc not only in themselves but as the components of brass are perhaps next in importance to iron in the world's industry to-day, and in 1918 they stood high on the list of war metals. A little more copper but less zinc was produced, and the values of both were lower, that of zinc falling about 25 per cent. The output of the war metals manganese and chromite, used in hardening steel, was greater than in any preceding year. Chromite increased 88 per cent. in quantity and 275 per cent. in value over 1917, and the increases in manganese ore were 136 and 100 per cent., respectively. Less gold and silver were mined than for many years. Though the price of silver rose from 81 cents an ounce in 1917 to nearly 97 cents in 1918, the increase was not enough to cover the increased cost of mining.

The output of building material—clay products, building stone, cement, lime, gypsum—showed a great decline.

The domestic production of potash in 1918 was 54,000 tons, an increase of 68 per cent. over the output in 1917.

AIR SPEED INDICATORS FOR DIRIGIBLES.

BY

THE AERONAUTICS STAFF, U. S. N.*

Construction Department, Washington Navy Yard.

To meet the war-time needs of the Navy for an air speed indicator for dirigibles, a speed nozzle was developed having the following required characteristics: water-proofness, accuracy in veering winds, and pressure intensity to admit of a rugged aneroid gauge. The development took two forms, both based on the well-known double-throat venturi, but made waterproof by shielding the smaller inner venturi from the direct impact of drops or spray. The first, being too heavy, was abandoned after field trial, in favor of the second which was lighter and easier to make.

PART I.

Figures 1 and 2 show the construction of the first instrument. In principle it consists of a double-throat venturi provided with

FIG. 1.

Hooded two-throat venturi No. 1.

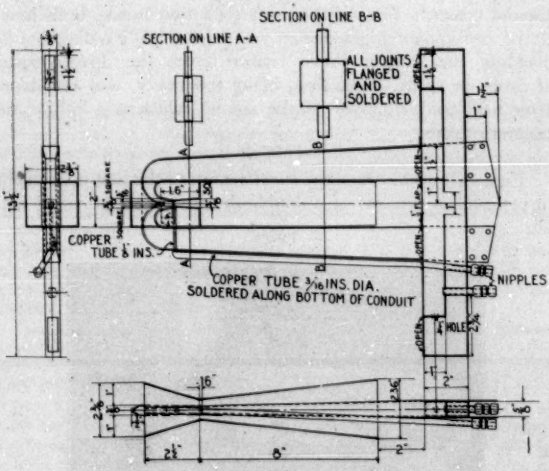
a dry air reservoir and flues leading therefrom to the interior of the smaller hooded venturi as shown.

The air rushing into the rectangular openings in the base of

* Communicated by A. F. Zahm, Ph.D.

the instrument comes almost to rest and generates nearly full impact pressure, while any water drops it may contain strike the rear wall, trickle down and run out the lower slot. Suitable baffle plates prevent any stray drops from reaching the flues. The dry air passing through these spacious flues enters the smaller cone of the interior venturi in symmetrical flow from opposite sides, and emerges rearward in the usual manner just at the throat of the larger surrounding venturi. Direct influx of the

FIG. 2.



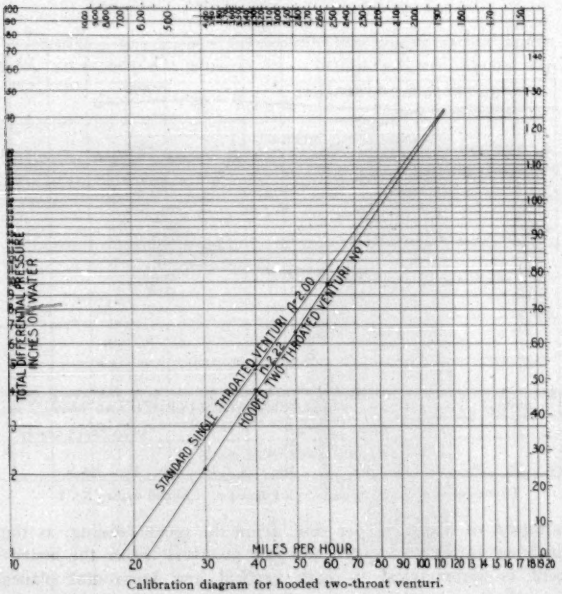
Plans for hooded two-throat venturi No. 1.

outer air into the smaller venturi is prevented by the stream-line hood shown protruding from the front. The large outer venturi is made in about the same proportions as an ordinary one, the throat area being about one-fourth that of the intake opening. The interior venturi also is made of the same proportions as an ordinary one. The impact pressure is collected from a nipple extending into the reservoir behind the baffle as shown. A tube leading from the throat of the smaller venturi collects the suction. The base of the instrument is provided with lugs for at-

tachment to a pole protruding from the car or rigging of the dirigible.

Figures 3 and 4 give the results of the wind tunnel tests; the first showing the differential pressure in terms of the wind speed; the second showing the fluctuation of the readings with variations of the wind direction in pitch and yaw.

FIG. 3.


TOTAL DIFFERENTIAL PRESSURE INCHES OF WATER
STANDARD SINGLE THROATED VENTURI P=2.00
TO 2.22
HOODED TWO THROATED VENTURI W.T.
MILES PER HOUR
10 20 30 40 50 60 70 80 90 100 110 120 13 14 15 16 17 18 19 20

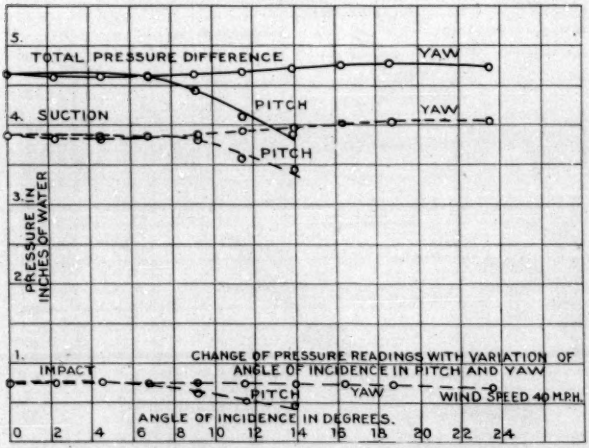
Calibration diagram for hooded two-throat venturi.

Figure 3 shows that the total differential pressure is 8.9 inches of water at 50 knots speed, and increases as the power 2.22 of the velocity. On the same diagram is shown the calibration curve for the Navy's standard single-throat venturi, whose readings increase as the square of the velocity. At the proposed dirigible speeds of 20 to 50 knots, the readings of the single-

throat instrument averages about 20 per cent. higher than those of the double-throat instrument. The latter could doubtless be made to read much higher than the single throater by suitably designing the flues; but this desideratum has been attained more simply in the second type mentioned.

Figure 4 presents the pressures in both the impact and suction nozzles—also their resultant pressure—for the various incidences in pitch and yaw. The resultant pressure diagram indicates a

FIG. 4.



Change of reading with change of wind direction, of hooded venturi No. 1

variation of about ½ per cent. from the mean reading, as the incidence varies 7° in pitch and yaw positive. Since the instrument is symmetrical in both vertical and horizontal planes through its axis the same readings should obtain for negative incidences. The lower dotted lines in the diagram indicate the impact pressure in the cistern, which is very approximately equal to $pv^2/2$. This taken from the total pressure differences gives the suction shown also by the dotted line.

When this first type of double-throat instrument was used on the school dirigible at Akron in July, 1917, together with a

Foxboro pressure gauge, it was found to give sufficiently strong and uniform readings. But its manufacture was not recommended; first, because the instrument was unnecessarily heavy; secondly, because it was expensive to make; thirdly, it was difficult to build sufficiently uniform to give the same results in different instruments; lastly, it was superseded by a lighter and more powerful instrument.

PART II.

Figures 5 and 6 show the construction of the second instrument. In principle it consists of an ordinary double-throat ven-

FIG. 5.

Hooded two-throat Venturi No. 2.

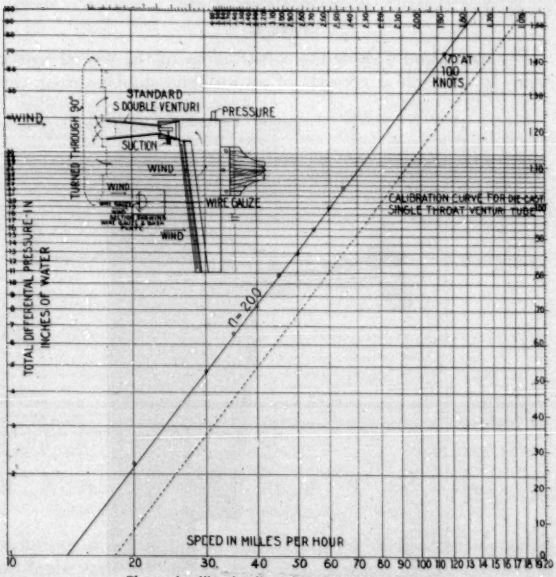
turi reversed in direction, its smaller end inserted in a dry air cistern, its larger end capped with a tee-tube which serves both to increase the suction and to shield the venturi from rain.

The wind driving into the rectangular slot of the cistern strikes against a wire screen designed to remove the rain drops; thence passes upward and forward through the regular venturi;

thence into the tee-pipe where it divides right and left, finally emerging from rectangular slots at the rear thereof.

Figure 6 presents the calibration curve of this instrument for all speeds from 20 to 70 M.P.H.; also that of the Navy's standard die-cast single-throat venturi, for comparison. In both,

FIG. 6.



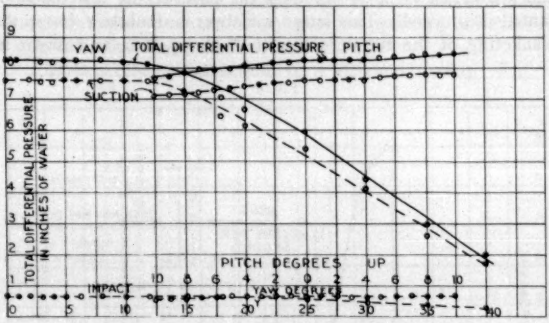
Plan and calibration line of hooded venturi No. 2.

the resultant pressure difference varies directly as the square of the velocity at all available speeds of the wind tunnel. At 100 knots the pressure difference is 41 inches of water for the single-throat instrument; 70 for the double-throat instrument with a rain-shield. Without the rain-shield the latter reading becomes 78, or nearly twice that for the single-throat instrument.

Figure 7 presents the resultant pressure difference at all in-

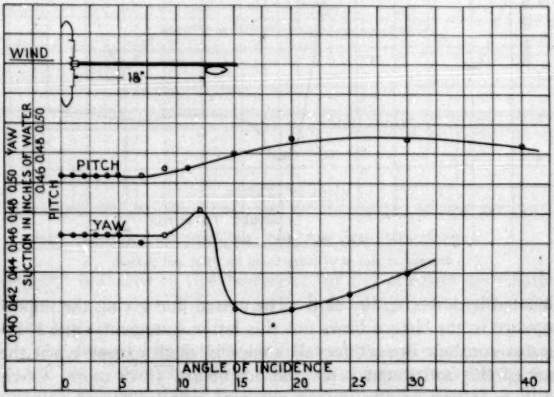
cidences in yaw from 0° to 40° positive; and the same may be expected for negative incidences, owing to symmetry; also the

FIG. 7.



Change of reading with change of wind direction, of hooded venturi No. 2.

FIG. 8.

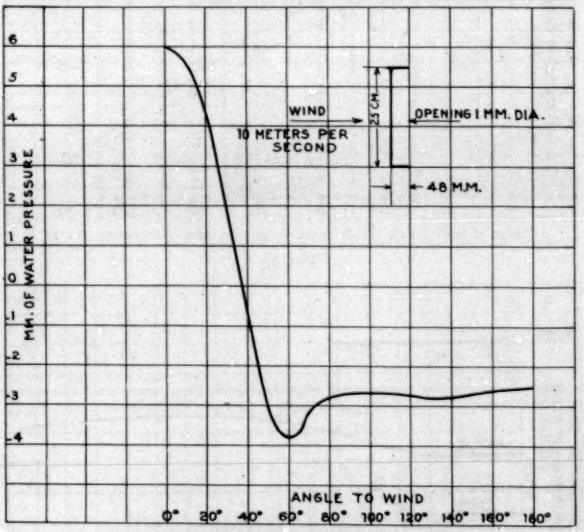


Suction at rear of tee-cap, at 40 miles an hour.

pressure differences for variations in pitch through 10° up and down. The yaw curve shows practically no variation in reading

for changes of incidence from 0° to 10°; the pitch curve shows an increase of pressure difference above the zero of slightly over one per cent. as the nose of the instrument is canted up 7°, and a decrease of 4 per cent. as the nose of the instrument is canted downward. This latter variation is doubtless due to the blanketing of the rectangular slot of the cistern, and might be

FIG. 9.



Suction at rear of cylinder found by Pirzi and Soldati.

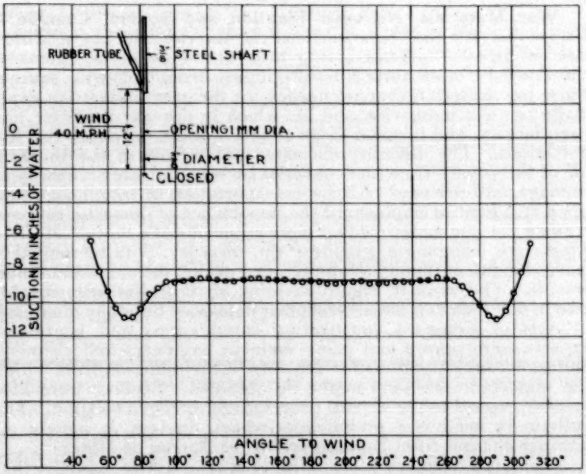
obviated by lowering the slot. The dotted line giving the impact pressure in the cistern bears out this latter statement, since it indicates constant impact for all angles in pitch except when the nose of the instrument is pointed downward 3° or more. Obviously a falling of the impact pressure entails loss of velocity through the venturi throat, and hence a loss of suction.

Figure 8 presents the suction readings of the tee-cap when placed alone in the wind tunnel, as shown in the sketch. This

diagram corroborates the results published by Doctors Finzi and Soldati in 1902 in their pamphlet "Esperimenti sulla Dinamica dei Fluidi," in which they show that the pressure distribution over the rear surface of a cylinder held squarely across the wind is approximately constant. They gave no readings, however, for a cylinder held oblique to the wind.

Figure 9 presents the data of Finzi and Soldati; Fig. 10

FIG. 10.



Suction behind cylinder 12 inches long by 3/8 inches diameter.

the data obtained in this research; for the pressure distribution over a meridian circle of a cylinder held squarely across the wind, as in the sketches.

The data of Fig. 9 were obtained with a cylinder 25 cm. long by 48 mm. diameter, held with the perforation facing first directly into the wind, then at various angles away from the wind all the way to 180°. The pressure is seen to be approximately constant over the rear portion of the cylinder to a distance of about 90° on either side of the exact rear.

Fig. 10 shows a still more exactly constant pressure over all the rear portion of the cylinder lying within about 80° on either side of the exact rear. The tube employed in this experiment was 12 inches long by 3/8 inch diameter, having a one millimetre opening for collection of the pressure.

A few of these speed nozzles were placed on dirigibles and kite-balloons during the war, and were found to give satisfactory service.

War Materials, Nitrogen Fixation and Sodium Cyanide.—Under this title the Department of the Interior, Bureau of Mines, has published a bulletin giving brief accounts of the efforts of the United States authorities to supply from domestic sources those raw materials that are needed for the great industries, especially the war industries, and for which in pre-war times we had been largely, and in some cases almost wholly, dependent on importations. The difficulty consisted not merely in the shutting off of the supply from the countries at war, but the shortness of tonnage, and the need of it for transportation of troops and supplies, still further emphasized the importance of domestic sources. Among the raw materials that were especially the subject of these efforts are manganese, graphite, tin, mercury, potash, tungsten, molybdenum, antimony, chromite, magnesite, mica, platinum and arsenic. The work in regard to some of these has been considered in this JOURNAL, but several other items may be briefly discussed.

Acts of Congress constituting commissions and boards of experts were passed and many eminent engineers and chemists undertook investigations for deposits of the minerals from which the important products might be obtained. Studies were also made in regard to the several processes of nitrogen fixation. The bulletin in hand is a preliminary report; in fact, is merely an advance chapter from Bulletin 178 of the Bureau of Mines.

One point of interest is worthy of special notice, namely, that the simple process of Bucher for the production of sodium cyanide has been shown to be practicable on the large scale.

In this connection notice should be given of the success of methods devised by L. S. Potsdamer for recovering valuable materials from gas-mass; that is, the mixture of ferric hydroxide and wood chips used for purifying illuminating gas. This accumulates a number of by-products, among which are tar, sulphur, thiocyanides, ammonium salts, ferro- and ferricyanides, double ammonium cyanides. Potsdamer devised a plan of recovering these substances and reviving the gas-mass so that it could be returned to the purifiers. The details of the methods will be found in *Jour. Ind. Eng. Chem.*, vol. xi, p. 769, 1919.

H. L.

EXPERIMENTS ON SO-CALLED "ROOT-BEERS."*

BY

CHARLES H. LaWALL, Ph.M., AND HENRY LEFFMANN, M.D.,

Members of the Institute.

THE adoption of drastic prohibitory methods in the United States has led to considerable discussion as to the relation between the alcohol content of standard beers and the household products such as "root-beer," "spruce-beer" and "mead." In judging of these latter forms it must be borne in mind that they are never used for true intoxication, but for their effervescent properties and the special flavors. The forms sold already bottled or drawn from fountains and used immediately are non-alcoholic, but the household production of an effervescing beverage can best be conducted by the fermentation of ordinary sugar with yeast. If such a mixture be made and kept at room temperature for a few days in an open vessel a notable amount of alcohol will be produced, but the liquid will be quite unfit for beverage purposes. To secure the effervescence the fermenting liquid must be bottled soon after the fermentation starts. As a rule the alcohol does not go above 1 per cent. after five or six days of fermentation under pressure. Statements in conflict with this having been made lately in connection with discussions of the bearing of the prohibition acts, we have deemed it worth while to repeat some experiments made by us some time ago, and submit figures. The alcohol was determined after distillation by the immersion refractometer, the reducing sugar gravimetrically by Fehling's solution. Polarimetric readings were also made. The mixture was made up in accordance with the directions given for one of the best known root-beers. The bottles were filled to within a small distance of the cork, and kept at room temperature during the latter part of July. The figures were obtained as follows:

	Reducing sugar	Alcohol by volume	Effervescence
When mixed	0.20	none	none
After 24 hours	7.7	0.20	very slight
After 3 days	6.9	0.39	slight
After 5 days	5.9	0.82	marked
After 8 days	5.2	1.18	marked

* Communicated by the Authors.

The polarimetric readings were irregular and showed fluctuations in which the minus readings increased while the reducing sugar decreased, due doubtless to the fact that the dextrose is more readily fermented by ordinary yeasts than levulose. A second series resulted as follows:

	Reducing sugar	Alcohol by volume
When mixed	0.19	none
In 24 hours	8.8	0.20
In 3 days	8.0	0.49
In 5 days	7.1	0.79
In 8 days	6.5	1.10

This series was kept at room temperature for three days then placed in a uniformly cool place (40° to 50° F.) for the remainder of the time.

No appreciable alcohol was obtained if the sample was placed in a cool room (40° to 50° F.) three hours after bottling, while the fermentation seemed to be active. The action seemed to be promptly checked, the alcohol not rising above 0.35 per cent. and the effervescence so slight as to make the product of little palatability.

These experiments, which agree with those obtained previously by us, indicate that when the product has attained sufficient sparkle to be agreeable, the alcohol ranges from 0.4 per cent. to 0.8 per cent. by volume. If the fermentation is allowed to go until more than 1.25 per cent. of alcohol is formed the liquid becomes unpalatable from the diminution of the sugar and the increase of yeast.

It also seems that in well-corked bottles the fermentation will not proceed so far as to produce 2 per cent. of alcohol by volume.

NOTES FROM THE U. S. BUREAU OF STANDARDS *

PHOTOELECTRIC SPECTROPHOTOMETRY BY THE NULL METHOD.[1]

By K. S. Gibson.

[ABSTRACT.]

In connection with the color standardization work of the National Bureau of Standards[2] it is desired to have available a number of independent methods of making spectrophotometric determinations, especially in the visible part of the spectrum; for it is generally admitted that the fundamental basis of color specification is spectrophotometry. To supplement the other methods at present in use at the Bureau and especially to overcome the well-known uncertainty of measurements by these other methods in the blue and violet, the author in 1917 was given the problem of developing a method for accurate and convenient photoelectric spectrophotometry suitable for routine determinations. This was especially desirable for the measurement of spectral transmission. The sensitive potassium-hydride photoelectric cells now on the market (made by Kunz) when used with an incandescent lamp and a glass dispersing prism, give a maximum response usually near 460 millimicrons, and are thus very suitable for the purpose. The making and assembling of apparatus was completed in April, 1918; and since that time it has been in continual use, being very satisfactory as to speed of operation, ease of keeping in working condition, and accuracy of measurement.

In the null method as used, two photoelectric cells and the proper batteries are connected in a sort of Wheatstone bridge arrangement,[3] with the electrometer as the indicator. Radiant energy from a 600-watt Mazda C moving-picture lamp, after dis-

* Communicated by the Director.

[1] Scientific Paper No. 349.

[2] I. G. Priest, "The Work of the National Bureau of Standards on the Establishment of Color Standards and Methods of Color Specifications." *Trans.* I. E. S., xiii, p. 38, 1918.

[3] F. K. Richtmyer, *Phys. Rev.*, (2) vi, p. 66, 1915.

persion through a Hilger constant deviation spectrometer, is incident upon one of the photoelectric cells. Radiant energy from a 14-watt Mazda C lamp is incident on the other photoelectric cell. The photoelectric currents generated in the two cells by the radiant energy from the two lamps tend to nullify each other so far as charging the electrometer is concerned; and by proper adjustments, the two currents may be made exactly equal, the zero motion of the spot of light from the electrometer indicating the balanced condition.

Measurements of the spectral transmission of a specimen are made by observing at any desired wave length the distances of the 600-watt lamp from the spectrometer slit necessary to obtain a balance of the electrometer with and without the specimen in position, all other factors such as slit widths, currents, etc., being kept constant. The ratio of the squares of these two distances respectively is the transmission. By this use of the null method all errors are eliminated, as well as the necessity of any kind of calibration, in connection with the relation between photoelectric current and incident radiant power, with the dark currents through the photoelectric cells, and with electrometer deflection methods.

The accuracy has been tested in various ways, chief of which are the measurements of rotating sectors of known transmission and comparisons with values obtained by other methods. It is considered that the uncertainties of values obtained from 410 to 550 millimicrons, inclusive, are not greater than 0.01 for any values of transmission between 0.00 and 1.00 and not greater than 0.003 for transmissions between 0.00 and 0.10. Beyond these wave lengths just given, as far as 390 and 600 millimicrons, inclusive, the uncertainties of measurement are not greatly increased, being at 390 and 600 not more than twice as great as throughout the better range. Measurements can be made from 380 to 650 millimicrons.

The apparatus has also been used for the measurement of spectral diffuse reflection relative to that of a standard such as magnesium carbonate, and is adapted to the measurement of the relative distribution of radiant power of two sources, to the measurement of fluorescence, and to extension into the ultraviolet if quartz parts instead of glass were used.

USE OF A MODIFIED ROSENHAIN FURNACE FOR THERMAL ANALYSIS.[4]

By H. Scott and J. R. Freeman, Jr.

[ABSTRACT.]

ROSENHAIN in 1915 published a description of a furnace for the thermal analysis of metals operating under a new principle. Instead of varying the temperature of the furnace containing the specimen under investigation, he heated the furnace so as to produce a uniform temperature gradient along its length and moved the specimen through the furnace. The present paper describes in detail some improvements in the design of his furnace and considers certain features of its operation. A simple method of mounting is described which gives highly satisfactory results using specimens of less than 2 grams weight.

THE HEAT TREATMENT OF DURALUMIN.[5]

By P. D. Merica, R. G. Waltenberg, and H. Scott.

[ABSTRACT.]

THE heat treatment of alloys of the type, duralumin, was investigated and the effect observed of variations in the heat treating conditions, such as quenching temperature, temperature of quenching bath and of aging or tempering, and time of aging upon the mechanical properties.

Conclusions are drawn relative to the best conditions for commercial heat treating practice for this alloy. The temperature of quenching should not be above that of the $CuAl_2$ aluminum eutectic, which is usually about 520° C., but should be as near to this as possible without danger of eutectic melting. The pieces should be held at this temperature from 10 to 20 minutes and quenched preferably in boiling water. The hardening may for most purposes best be produced by aging for about 5 days at 100° C.

A theory of the mechanism of hardening of duralumin during aging after quenching from higher temperatures was de-

[4] Scientific Paper No. 348.
[5] Scientific Paper No. 347.

veloped which is based upon the decreasing solubility of the compound $CuAl_2$ in solid solution in aluminum with decreasing temperatures from 520° to ordinary temperatures. It is believed that the precipitation of excess $CuAl_2$, which is suppressed by quenching, proceeds during aging, the precipitation taking place in very highly dispersed form. The hardening is due to the formation of this highly dispersed precipitate.

According to this theory the hardening of duralumin during aging or tempering after quenching presents a very close analogy with that of steel, and the evidence in support of the theory is of the same nature and of approximately the same competence as that in support of the prevailing theory of the hardening of steel.

CONSTANT TEMPERATURE STILL HEAD FOR LIGHT OIL FRACTIONATION.*

By Frederick M. Washburn.

[ABSTRACT.]

THIS paper represents the results of a brief search for an improved method for the determination of benzine, toluene, and solvent naphtha in light oil.

The three types of methods generally in use for the fractionation of light oil for the determination of benzine, toluene, and solvent naphtha are discussed. An apparatus which is an improvement on the dephlegmator of the Wilson and Roberts still is described, and the details of its operation are given. The apparatus is easily and inexpensively constructed, and requires no greater attention or time than others used. Exceptionally large volumes of "pure" fractions are obtained which have a very small boiling point range, showing that they contain only negligible amounts of impurities. Almost all of each of the components of the mixture distilled is obtained in practically the pure state, since the volumes of each of the intermediate fractions are only about 1.5 per cent. of the volume taken for distillation. The composition of each of the intermediate fractions is actually determined, and the error introduced by this determination is small,

* Technologic Paper No. 140.

since it is applied to only small volumes. The apparatus works well on mixtures containing widely varying percentages of benzine, toluene, and solvent naphtha.

KITCHEN CARD OF HOUSEHOLD WEIGHTS AND MEASURES.[1]

[ABSTRACT.]

The tables of weights and measures most useful for household purposes are being published on a card designed to be hung in the kitchen where it can be easily referred to.

The card contains tables of liquid measure, dry measure, avoirdupois weights, linear measure, square measure and cubic measure. Rules are given for calculating the area of the rectangle, circle, and cylinder and the volumes of some common geometrical figures. The approximate weights of many commodities used in the household and particularly in the kitchen, are included.

NOTICE REGARDING BIBLIOGRAPHY OF HELIUM LITERATURE.[2]

By E. R. Weaver.

[ABSTRACT.]

The year 1918 marks the beginning of a new era in the history and use of helium. Before that time only a few litres of the gas had been collected and the cost per litre was enormous. During the war the development of great factionating plants capable of separating from natural gas a sufficient quantity of helium to supply a fleet of airships has aroused the keen interest, not only of engineers and scientists, but also of the general public in the unique properties of this gas.

A bibliography of the scientific literature relating to helium was prepared for use during the war and is now published as Circular 81 of the Bureau of Standards. This bibliography will serve as a guide to the subject and stimulate research by pointing out the sources of scientific data concerning helium.

[1] Miscellaneous Publication No. 39.

[2] Circular 81.

Related papers are grouped together in their chronological order, making the bibliography, in effect, a brief outline history of the subject.

REFLECTION POWER OF STELLITE AND LACQUERED SILVER.[9]

By W. W. Coblentz and H. Kahler.

[ABSTRACT.]

IT IS shown that the reflectivity of stellite varies somewhat in the visible spectrum, depending upon the homogeneity and no doubt upon the exact composition of the alloy.

Data are given on the reflecting power of lacquered silver mirrors, before and after exposure to ultra-violet light. It is shown that owing to photochemical action in the lacquer, the silver is turned brown in color, thus reducing its reflecting power.

THE LOCATION OF FLAWS IN RIFLE BARREL STEEL BY MAGNETIC ANALYSIS.[10]

By R. L. Sanford and Wm. B. Kouwenhoven.

[ABSTRACT.]

THIS paper describes an investigation which was undertaken for the purpose of determining whether an application of magnetic analysis was practicable for the detection of flaws in rifle barrel steel. By means of apparatus especially constructed for the purpose a large number of bars were explored for magnetic uniformity along their length. In spite of the fact that these bars were taken from material which had previously been rejected as the result of drilling tests, not one was found which contained a pipe. The results obtained, however, demonstrated that the method is amply sensitive to detect and locate flaws. Further study is necessary to determine to what degree the sensitivity of the apparatus should be reduced in order not to cause the rejection of material which is satisfactory for all practical purposes and also to determine the type and magnitude of the effect which

[9] Scientific Paper No. 342.
[10] Scientific Paper No. 343.

will be produced by a pipe. For this reason the work is being continued by the Winchester Repeating Arms Company who coöperated in the investigation and at whose plant the apparatus has been installed.

MEASUREMENTS OF WAVE LENGTHS IN THE SPECTRA OF KRYPTON AND XENON.[11]

By Paul W. Merrill.

[ABSTRACT.]

This paper records photographic measurements of wave lengths in the spectra of krypton and xenon, principally in the red and infra-red. In krypton 37 new lines were measured between 6576 A and 8928 A, in xenon 52 lines between 6318 A and 9162 A. In this region there are numerous strong lines which are probably among the most important in the spectra of these elements. Notable among these are xenon lines at 8231 and 8280. These and other lines may be of value as wave-length standards in the infra-red.

Attention is called to a probable analogy between the spectra of the rare gases neon, argon, krypton, and xenon which this investigation has brought to light.

RECOMMENDED SPECIFICATION FOR LINSEED OIL, RAW, REFINED AND BOILED.[12]

[ABSTRACT.]

Prepared and recommended by the U. S. Interdepartmental Committee on Standardizing Paint Specifications, April 16, 1919. This Committee was appointed at the suggestion of the Secretary of Commerce and consisted of representatives of the War, Navy, Agriculture, Interior, Post Office, Treasury, and Commerce Departments, the Railroad Administration, the Panama Canal, and the War Service Committee of the Paint Manufacturers' Association of the United States. The Committee submitted a preliminary draft of this specification to more than 300

[11] Scientific Paper No. 345.
[12] Circular 82.

representatives of the paint and varnish industries, including all of the large manufacturers of linseed oil, and gave careful consideration to the large number of replies received in time.

OXYGEN CONTENT BY THE LEDEBUR METHOD OF ACID BESSEMER STEELS DEOXIDIZED IN VARIOUS WAYS.[13]

By J. R. Cain and Earl Pettijohn.

[ABSTRACT.]

IT IS shown that the Ledebur method for determining oxygen in steels indicates no marked difference in oxygen contents of steels practically identical as to chemical composition and heat treatment, but made by different deoxidation treatments. Some differences in physical properties of such steels are shown.

THE BEHAVIOR OF WROUGHT MANGANESE BRONZE EXPOSED TO CORROSION, WHILE UNDER TENSILE STRESS.[14]

By P. D. Merica and R. W. Woodward.

[ABSTRACT.]

SPECIMENS of wrought manganese bronze rods were exposed in special steel test frames to corrosion in water and moist air while at the same time under tensile stress with the object of determining the maximum safe stresses for this material under these conditions. While the period of exposure was only two years and the initial condition of the best bars not normal in that the initial stresses had been removed by a low temperature anneal, the results of the tests are capable of some general interpretations.

None of the test bars fractured within this period under stresses below the proportional limit, and four of the bars withstood corrosion during this period under stresses which produced slight yielding and permanent set when first applied.

Fracture did not occur under stresses less than 35,000 pounds per square inch.

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[13] Scientific Paper No. 346.

[14] Technologic Paper No. 135.

One specimen with a proportional limit of 35,000 pounds per square inch fractured under a tensile stress of approximately 40,000 pounds per square inch, and another with a proportional limit of 42,500 pounds per square inch, under a stress of approximately 43,000 pounds per square inch.

APPLICATION OF THE INTERFEROMETER TO GAS ANALYSIS.[15]

By Junius David Edwards.

[ABSTRACT.]

ONE of the most useful of the physical methods applicable to gas analysis is that of gas interferometry. By the application of a new method of calibration, previously described in Scientific Paper No. 316, the use of the gas interferometer has been simplified and extended. The relation betwen the refractivities of the gases and the indications of the interferometer is discussed for various typical cases and illustrative calculations given. The determination of helium in a mixture of gases is one case of interest which is discussed. It is of importance because of the scarcity of analytical methods for determining helium. Other cases discussed are the effects of variations in the composition of air where it is a component of mixtures under test, the analysis of flue gases, the relative sensitivity of the interferometer for different gases and points about the operation of the interferometer.

EXPERIMENTAL-RETORT TESTS OF ORIENT COAL.[16]

By R. S. McBride and I. V. Brumbaugh.

[ABSTRACT.]

THE Bureau of Standards in connection with coke-oven investigations found it desirable to carry out a short series of experimental-retort tests of Illinois coal to determine the influence of temperature of coking upon the characteristics of the

[15] Technologic Paper No. 131.
[16] Technical Paper No. 134.

coke and quantity and quality of gas produced. This work was done at the Sparrows' Point plant of the Bethlehem Steel Company, where apparatus was placed at the disposal of the Bureau through the courtesy of the engineers in charge.

The gas produced at high temperatures was much greater in volume but lower in heating value than that produced at the lower temperatures, not only because of more complete elimination of volatile matter from the coal, but also as a result of the greater decomposition of the heavier volatile matter into gaseous constituents at the higher temperature. At the lower temperatures the coke was very inferior to that produced at the higher temperatures, but in no case was the temperature maintained as high as is generally used in coke-oven practice.

Detailed results for the series of five tests are given, and a separate series of results on other coals is included, furnishing data given to the Bureau by the Steel Company covering tests made with the same apparatus during the preceding two years.

AIRPLANE ANTENNA CONSTANTS.[17]

By J. M. Cork.

[ABSTRACT.]

THIS paper contains data observed by the writer while an officer in the Signal Corps, U. S. Army, and is published by permission of the Chief Signal Officer.

The purpose of this work was to devise a method for measuring airplane antenna constants (*i.e.*, capacity, inductance, natural wave length) under conditions of actual flight; and to use this method to obtain data on various forms of fixed and trailing wires.

The principle of the method involves a continuous wave oscillator feeding directly into the antenna and substituting for the antenna a variable calibrated condenser and adjusting for the same wave length as with the antenna in the oscillating circuit. The result obtained is the effective capacity of the antenna. Having found this, a variable calibrated non-inductive resistance is varied until the D. C. component of the plate current reads the

[17] Scientific Paper No. 341.

same as for the real antenna. This gives the effective antenna resistance. Knowing the effective values of capacity at various wave lengths, the true capacity, inductance and natural wave length are readily found.

By this method results were obtained with various forms of fixed wires, and one, two and four trailing wires of various lengths are summarized.

A method for finding the directional transmitting effect of various antennas is also described. In order to compare the amounts of energy received, a detector tube with a three-stage audio amplifier is used. A transformer is placed in the plate circuit of the last amplifier tube, the secondary of which is connected to the heater coil of a thermocouple connected to a D.C. micro ammeter. This when calibrated is free from many of the uncertainties of the ordinary audibility meter. A typical directional curve of the trailing wire antenna is included in the paper.

EFFECTS OF GLUCOSE AND SALTS ON THE WEARING QUALITY OF SOLE LEATHER.*

By P. L. Wormeley, R. C. Bowker, R. W. Hart, L. M. Whitmore, and J. B. Churchill.

[ABSTRACT.]

TECHNOLOGIC Paper No. 138, Bureau of Standards, on the "Effects of Glucose and Salts on the Wearing Quality of Sole Leather," contains a description of the methods used and the results obtained from the first of a series of tests to be made on this subject. Four brands of leather were tested; two tannages to which very small amounts of glucose and salts were added and two tannages to which larger amounts of these materials were added. The experimental work consisted of actual service tests on shoes, tests on a laboratory wearing machine, water absorption tests and complete chemical analyses of the original and worn leathers. Results are presented which show the variation in wear of the different leathers, the variation in wear of soles cut from different locations on the hide, the water-absorption qualities of the leathers and the variation in chemical composition

* Technologic Paper No. 138.

of the leathers in different parts of the hide for both the new and worn soles. From the results of the test there is no indication that the addition of glucose and salts is either beneficial or detrimental to the durability of the leather and it is conclusively shown that the greater part of the added glucose and salts was lost from the leather during wear while the other water-soluble materials appear to be retained in the leather.

A COMPARISON OF THE HEAT-INSULATING PROPERTIES OF MATERIALS USED IN FIRE-RESISTIVE CONSTRUCTION.*

By Walter A. Hull.

[ABSTRACT.]

A COMPARISON of the relative efficiencies of the different classes of materials commonly used in fire-resistive construction to protect load-bearing members from the heat of an accidental fire has recently been made in an investigation by the Bureau of Standards. All materials were made up into specimens in the form of solid cylinders, 8 inches in diameter and 16 inches long. These were tested separately in a specially designed gas fired furnace, the furnace temperature being raised gradually to 927° C. in ninety minutes and held at approximately that point for two hours.

Temperatures were measured by means of thermocouples at four points in the interior of each specimen. The temperatures were recorded throughout each test, giving information as to the rate of heat penetration in each sort of material. The condition of specimens after the test gave information as to the extent of deterioration which took place in each kind of material due to the heat.

The investigation included three clays that are used for the manufacture of terra cotta tile, concretes from two mixtures, 1:2:4 and 1:3:6, with a number of different coarse aggregates, gypsums of several different mixtures from raw materials from different parts of the country, and lime mortar.

The most rapid heat penetrations were found in the denser specimens of the burned clays. A porous burning clay, light burned, heated through as slowly as most of the concretes and

* Technologic Paper No. 130.

much less rapidly than the denser, harder burned clays. No clay specimens of the porosity of terra cotta lumber produced by the addition of sawdust were included.

Among the concretes, those with a coarse aggregate of gravel and those from bituminous cinders containing a large proportion of combustible material heated through most rapidly. Concretes from clean bituminous cinders, trap rock and blast furnace slag heated through more slowly, while the limestone concrete specimens heated through somewhat more slowly than any of the others. The less rapid temperature progress in the limestone concrete specimens may be attributed to the heat absorbed by the decarbonation of the limestone in a thin layer of concrete next to the exposed surface and to the fact that the resulting calcium oxide was presumably a better heat insulator than the original stone.

Gypsum specimens showed long temperature lags at about 105° C., due to dehydration of the material. Temperature progress through the gypsum specimens was much slower than through the clays and concretes. The densest, heaviest gypsum mixture heated through slightly more slowly than the lighter ones, due presumably to the great heat-absorbing capacity of the heavier specimens.

Lime mortar made a favorable showing, thermally, as compared with the concretes.

All the concretes were weak after test. Limestone concretes retained somewhat more strength than those from other coarse aggregates, though the cinder concretes were tougher than the others. The gravel concretes were the weakest of all after test. This is attributed largely to the fact that the gravel was made up of quartz and quartzose materials, as it is known that the expansion behavior of quartz, including the sudden volume increase as the inversion point at 575° C. is approached, is especially conducive to destructive strains. The gypsum and the lime mortar retained very little strength after the test.

The results are consistent with earlier information as to the relative termal qualities of the three classes of materials, burned clays, concretes, and gypsums, but are favorable to members of each class; namely, porous clays, limestone concretes, and the denser gypsums. The showing of gravel concrete both thermally and in respect to loss of strength was unfavorable.

THE DETERMINATION OF FREE CARBON IN RUBBER GOODS.*

By A. H. Smith and S. W. Epstein.

[ABSTRACT.]

THE method is briefly as follows: Extract a 1-g sample for six hours with acetone and then for three hours with chloroform or carbon bisulphide. Transfer the sample to a 250-cm³ beaker and heat on the steam bath until it no longer smells of chloroform. Add a few cm³ of hot concentrated nitric acid and allow to stand in the cold for about 10 minutes. Add 50 cm³ more of hot concentrated nitric acid, taking care to wash down the sides of the beaker. Heat on the steam bath for about one hour or until all bubbles or foam disappear from the surface. Pour the liquid, while hot, into a Gooch crucible containing a thick pad of ignited asbestos. Filter by slowly applying gentle suction and wash well with hot concentrated nitric acid. Empty the filter flask and wash the filter alternately with acetone and benzol until the filtrate is colorless. Next wash it well with a hot 15 per cent. solution of sodium hydroxide. Test for the presence of lead by running some warm ammonium acetate solution, containing an excess of ammonium hydroxide, through the pad into a solution of sodium chromate. If a yellow precipitate forms, the pad must be washed with the ammonium-acetate solution until the washings no longer precipitate the sodium-chromate solution. Next wash the residue a few times with hot concentrated hydrochloric acid and finally with warm 5 per cent. hydrochloric-acid solution. Remove the crucible from the funnel, taking care that the outside is clean, and dry it in an air bath for one and one-half hours at 150° C. Weigh, burn off the carbon at a dull-red heat, and reweigh. The difference in weight represents approximately 105 per cent. of the carbon originally present in the form of lampblack or gas black. It is recommended that 0.5-g samples be taken for compounds containing over 10 per cent. of free carbon and 1-g samples for compounds containing less than this amount. It is also unnecessary to extract high-grade compounds.

After a brief review of the literature, a discussion is given of the difficulties encountered in the use of the nitric acid method.

* Technologic Paper No. 136.

It is shown that nitric acid attacks the carbon and gives an insoluble compound, with the result that a factor of 1.05 must be used. An experiment is outlined which is taken to prove that bituminous matter is all removed by the treatment indicated. The effects of various mineral constituents are discussed and methods are outlined for their removal.

The authors conclude that, though the attack of nitric acid on carbon makes a very accurate determination impossible, the error caused thereby when the factor 1.05 is used is sufficiently small to justify the use of this method at the present time as a routine one in the rubber laboratory.

THE MECHANICAL PROPERTIES AND RESISTANCE TO CORROSION OF ROLLED LIGHT ALLOYS OF ALUMINUM AND MAGNESIUM WITH COPPER, WITH NICKEL AND WITH MANGANESE.[1]

By P. D. Merica, R. G. Waltenberg, and A. N. Finn.

[ABSTRACT.]

LIGHT aluminum alloys of several compositions belonging to each of the three ternary series: aluminum-magnesium-copper, aluminum-magnesium-manganese, and aluminum-magnesium-nickel, were rolled out into sheet and tested in tension as cold-rolled, after annealing, and after heat treatment, consisting of quenching from about 500° C. and aging at ordinary temperature.

The alloys of the aluminum-magnesium-copper series were superior in all conditions to those of the other series, in respect to tensile properties.

The tensile properties of the aluminum-magnesium-copper series may be much improved by appropriate heat treatment. The alloys of the aluminum-magnesium-nickel series are also improved by heat treatment, but not in the same degree as the former series. The alloys of the aluminum-magnesium-manganese series are not improved by heat treatment.

Samples of representative compositions of each series were exposed to corrosion in the salt spray test, and the appearance of samples observed after one and after two months' exposure to the action of the salt spray.

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[1] Technologic Paper No. 132.

The alloys of the aluminum-magnesium series resisted corrosion in general better than those of the other series, and this agrees with other experience in the corrosion of such alloys. The heat-treated specimens of the aluminum-magnesium-copper series were, however, but little inferior to those of the manganese series in their resistance to corrosion; the annealed and the cold-rolled samples of that series were the least resistant to corrosion of any of the alloys tested. Hard-rolled commercial aluminum corroded much more than any of the alloys. Annealed aluminum was more resistant to corrosion than the hard-rolled aluminum but did not compare favorably with most of the alloys.

TESTS OF FLEXIBLE GAS TUBING.[*]

By R. S. McBride and Walter M. Berry.

[ABSTRACT.]

A LARGE number of accidents in recent years, resulting in loss of life and property, have been caused by the poor quality and the improper and careless use of flexible gas tubing. Some states have prohibited the use of such tubing and there has been increasing agitation, by fire prevention authorities and others who are active in the cause of safety, for greater restrictions in the use of this material.

The Bureau of Standards has made tests on about thirty typical varieties of tubing and the tests show that much of the material on the market is made of paper, covered with some tar compound, and in whole of such dangerous construction that some steps should be taken immediately to prohibit the use of such tubing.

There are several varieties of tubing that, if used with ordinary care, can be considered safe for all ordinary use; and the life of the better varieties is so much greater that it is really cheaper in the end. Much of the danger in gas tubings comes from the poor grade of rubber end pieces, and the tests showed that there was as great a variation in the quality of these as in the quality of the tubings themselves. One high-grade sample which probably would not increase the cost of the tubing more

[*] Technical Paper No. 133.

than a few cents showed ten times the tensile strength and percentage elongation at break of that of the cheaper samples.

The experiments cover tests of transverse strength; tensile strength; pull required to remove end piece; tightness when straight, bent, or twisted; pressure loss; flexibility; heating test; rubber slip ends; and detachable metal end-pieces.

The American gas engineers have realized that the interests of the gas industry require the production of a safe gas tubing and as a result the National Commercial Gas Association and the American Gas Institute working in conjunction formulated a set of tentative specifications and presented them at the 1916 meeting of these societies. The tests reported by the Bureau have been made along the lines suggested by these proposals and the results are discussed to show where, in the judgment of the Bureau, these tentative specifications are too rigid, where too lenient, and to what extent these specifications fulfill their purpose, *i.e.*, of providing safe, convenient, and commercially suitable tubing.

Before final adoption of specifications either by the gas associations or the Bureau of Standards for the National Gas Safety Code, it is anticipated that coöperation in this work will enable the Bureau and the associations to formulate mutually satisfactory standards so that the specifications which are finally adopted may be identical. As a step toward this highly desirable end, a report of these tests has been prepared and is now presented to bring out the further discussion which is essential before more definite recommendations are made.

THE SPECTRAL PHOTOELECTRIC SENSITIVITY OF SILVER SULPHIDE AND SEVERAL OTHER SUBSTANCES.[28]

By W. W. Coblentz and H. Kahler.

[ABSTRACT.]

THIS paper gives data on the change in the electrical resistance of the sulphides of silver and of bismuth, when exposed to radiations of wave lengths extending from 0.6μ to 3μ. Measurements were made also upon galena, cylindrite, pyrites and jamesonite,

[28] Scientific Paper No. 344.

which, however, did not show photoelectrical sensitivity for the highest spectral radiation intensities available.

Both the natural mineral, acanthite, Ag_2S and a laboratory preparation were examined. The latter material, which was hammered into a thin plate, was found insensitive photoelectrically, at room temperature. But at $-157°$ C. a sharp maximum of photoelectrical sensitivity was observed for radiations of wave length $\lambda = 1.05\mu$.

At room temperatures the natural crystalline material differs from other photoelectrically sensitive substances studied, in that the photoelectric response becomes fatigued and after an exposure of 2 to 3 seconds to light, the positive resistance change begins to be effective. On removing the light stimulus, the galvanometer gives a negative deflection, which in the course of a few minutes returns to the original zero scale reading. In other words, the change in resistance of the crystal when exposed to radiation is first negative, then positive, the resultant change being negative and roughly one-fourth the original change. At low temperatures, $-158°$ C., this polarization phenomenon disappears, and the response to radiation is the same as that of other substances, *e.g.*, selenium. The spectral photoelectric sensitivity curve of crystalline silver sulphide, acanthite, at room temperature is conspicuous for its high sensitivity in the region of the spectrum, extending from 0.6μ to 1.2μ, followed by a maximum at about 1.35μ. Lowering the temperature to $-158°$ C. greatly increases the photoelectrical sensitivity of acanthite and produces a quite symmetrical curve with a maximum at 1.2μ.

Increasing the intensity of the exciting radiations shifts the maximum of the photoelectrical sensitivity curve toward the long wave lengths.

There is no simple law governing the variation in the photoelectric response in silver sulphide with variation in intensity of the radiation stimulus.

Mechanical working (hammering into a thin plate) appears to lower the intrinsic photoelectrical sensitivity of acanthite and changes the position of the maximum of spectral sensitivity.

A spectral photoelectric sensitivity curve of bismuthinite, Bi_2S_3, was obtained at $-166°$ C. There are maxima of sensitivity at 0.64μ, and 1.08μ, respectively.

NOTES FROM THE U. S. BUREAU OF CHEMISTRY.*

THE COMPOSITION AND BAKING VALUE OF THE DIFFERENT SIZED PARTICLES OF FLOUR.[1]

By J. A. LeClerc, H. L. Wessling, L. H. Bailey, and W. O. Gordon.

[ABSTRACT.]

AN investigation was conducted with 14 samples of flour to determine the composition and baking value of the different sized particles. Almost one-fourth of the flour, all of which was passed through a series of bolting silks number 15, 18, 20 and 21, remained on the No. 15 XX silk, while one-third passed through the No. 21. Upon analyzing and baking each separate, it was found that the quality of the coarse and very fine separates is inferior to that of the intermediate portion. The very fine separate is by far the poorest, both in quantity and quality of the gluten, as well as in the quality of the bread it produces. A superior quality of bread was secured when only the intermediate separates were selected for the baking.

THE CRYSTALLOGRAPHY OF MORPHINE AND CERTAIN OF ITS DERIVATIVES.[2]

By Edgar T. Wherry and Elias Yanovsky.

[ABSTRACT.]

DETAILED crystallographic measurements and partial optical observations have been made on morphine monohydrate, codeine (morphine methyl ester), in both anhydrous and monohydrate forms, codethyline (morphine ethyl ester), monohydrate, and heroine (diacetyl morphine), anhydrous. All crystallize in the rhombic system, and on calculating the topic axes it is found that there are definite relations between them, it being possible to

* Communicated by the Chief of the Bureau.

[1] Published in the *Operative Miller*, vol. xxiv, No. 8, August, 1919.

[2] Published in *J. Washington Acad. Sci.*, vol. ix, No. 16.

recognize the direction in the crystal in which addition of the CH_2 group, the H_2O of crystallization, and the acetyl groups takes place. The substances are too soluble in the liquids ordinarily used for the study of optical properties by the immersion method to permit accurate determinations of their refractive indices to be made.

Producer Gas for Internal Combustion Engines.—The shortage of gasoline in Great Britain during the war led to extensive investigations for substitutes, among which were ordinary illuminating gas and producer gas. A report has just been published by the committee appointed to investigate the subject, an abstract of which is given in the *English Mechanic* (vol. cx, p. 31, 1919). The committee considers that gas traction is as safe as the ordinary form even when unprotected and exposed flexible containers are used. In its newer forms it is well worth consideration as an alternative of electric, gasoline or steam traction. Common city gas is estimated to be equivalent to gasoline in the proportion of 250 cubic feet of the former to 1 gallon of the latter. Presumably, these being English data, the city gas is more largely hydrocarbons than much of the gas supplied to American cities, and the gallon may be the imperial gallon of ten avoirdupois pounds. It is recommended, however, that gas-bags should be replaced by rigid or semi-rigid containers, whenever a compression plant can be installed, but it is not considered advisable that any existing plant for gas-traction should be abandoned at this time. Figures on cost of compressing, etc., are given, but they are obviously of no value in this country.

In a supplementary article (E. M., same volume, page 45) Mr. David J. Smith, who made many of the experiments in the matter, states that it is possible to run satisfactorily in competition with gasoline motor vehicles by producing gas made on the vehicle, using anthracite, coke or charcoal. He states that the cost of running a truck with anthracite at 50 shillings ($11) per ton was equivalent to gasoline at 5.4 pence (11 cents) per gallon, the commercial rate of the gasoline being taken at 2s.6d. (about 60 cents) per gallon. He claims that a producer can be made according to his designs that will occupy no loading room on the truck, and free access to the equipment is secured. In case of trucks the weight of the equipment for producing the gas does not exceed 2 per cent. of the weight of the loaded vehicle. The method is applicable to boats and tractors, the small size of the plant rendering it suitable for applications to which formerly producer gas apparatus could not be applied.

H. L.

NOTES FROM THE U. S. BUREAU OF MINES *

Aluminum Oxidation.—The oxidation of aluminum is being studied at the Pittsburgh station. A sample of aluminum powder was analyzed for oxide by all the methods that are in use, for the purpose of comparing and perfecting them. Comparative figures were obtained for the oxygen content of an aluminum sample as determined by the sodium hydroxide and iodine methods. The latter shows consistently a much higher oxide value than the former, and both methods show a steady decrease in the amount of oxide with increase in the time of contact between the reagent and the sample. Further work is in progress.

Neumann Bands.—Tests for the Neumann Band Committee of the National Research Council have progressed at the Explosives Experiment Station under the direction of S. P. Howell, a member of that committee, and four tests in triplicate were made on steel disks prepared from cold rolled steel shafting, carbon less than 0.15. Each set of disks was distorted by explosion of explosives in contact with them, the explosives having rates of detonation of 5716, 4470, 3190 and 1523 metres per second. These disks will be microscopically examined by Mr. F. B. Foley, of the Bureau of Mines, also a member of this Committee, now at Bedford Hills, New York. The object of the investigation is to determine what effect the velocity of deformation or rupture of steel has upon the number or character of Neumann bands produced in the steel.

Lead Density.—It appears that the density of lead is decreased materially when distorted quickly, as by an explosion, as shown by recent tests made at the Explosives Experiment Station of the Bureau of Mines, under the direction of S. P. Howell. When small lead cylinders having an average density of 11.403 were distorted by an explosive having a rate of detonation of 3341 metres per second, its density was reduced to 10.996, and when distorted by an explosive whose rate of detonation was 3035 metres per second, the density was reduced from 11.376 to 11.156, and when the rate of detonation of the explosive was

* Communicated by the Director.

2835 metres per second, the density of the lead was reduced from 11.383 to 11.228. It is believed, that this reduction in density is concomitant to the formation of minute voids in the lead, and further investigation is required to determine this. This confirms tests made in the winter 1888–1889 at the Massachusetts Institute of Technology by Robert C. Williams and J. C. Seager, and reported by Frederick W. Clark in the *Transactions of the American Institute of Mining Engineers*, vol. xviii, pp. 526, 527, in which the specific gravity of lead cylinders before firing was 11.51 and after firing a charge of explosive in the borehole in the lead cylinder, the specific gravity was 11.268.

Production of Ammonia by Synthesis.—The economic production of ammonia from nitrogen and hydrogen directly will be a valuable addition to chemical industries. The Haber-Le Rossignol process has been brought to a high degree of perfection, and it is claimed that Germany relied almost entirely on it for the supply of nitrogen compounds during the war. American chemists have given considerable attention to it also. Briner and Baerfuss, in a lengthy paper in *Jour. d. Chim. Phys.* (vol. xvii, p. 71, 1919), give results of experiments on the union of nitrogen and hydrogen under the influence of the electric arc, the gases being under diminished pressure. They summarize the results as follows:

At pressures of not over 150 mm. of mercury, the arc assumes a sheath form, the length being, of course, dependent on the current strength and the pressure. This form has a powerful influence on the determination of the synthesis. Changes of pressure and of the metal used for electrodes affect the output of ammonia. The experimenters believe that the combination takes place because the molecules of the two gases are dissociated into the constituent atoms in the region of the arc. The reduced pressure favors this action and, therefore, contributes materially to the production of ammonia. The dissociation may be due to the heat alone, or result from more complex conditions. The condition described by Strutt, *Proc. Roy. Soc.* (vol. lxxxv, p. 219, 1911, *et seq.*), namely, the formation of allotropic nitrogen of high activity, was not observed in these experiments. The best results should theoretically be obtained by the use of a mixture of the elements in the proportion in which they exist in the compound, *i.e.*, N to H_3, but as both elements are not equally activated by the arc, it is likely that in practice an excess of nitrogen should be present as this is less activated than hydrogen.

H. L.

THE FRANKLIN INSTITUTE.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of the Stated Meeting held Wednesday, September 3, 1919.)

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, September 3, 1919.

MR. BENJAMIN FRANKLIN *in the Chair.*

The following report was presented for final action:

No. 2729: Simplex Fluid Meter. Action deferred.

R. B. OWENS,
Secretary.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, September 10, 1919.)

RESIDENT.

MR. JOSEPH A. DOANE, Chemist, The Pennsylvania Salt Manufacturing Company, and for mail, 1243 Sedgley Avenue, Philadelphia, Pennsylvania.

MR. GUY D. FERNANDEZ, Electrical Engineer, 217 Atlantic Avenue, Camden, New Jersey, and for mail, Haddon Heights, New Jersey.

MR. HOWARD S. GRAHAM, Banker, 435 Chestnut Street, and for mail, 205 West Chestnut Avenue, Chestnut Hill, Philadelphia, Pennsylvania.

MR. H. CONRAD MEYER, Vice-President, Foote Mineral Company, 107 North 19th Street, and for mail, 70 East Penn Street, Germantown, Philadelphia, Pennsylvania.

MR. WILLIAM TIDDY, Chief Chemist, Rainey-Wood Coke Company, Swedeland, Pennsylvania.

NON-RESIDENT.

MR. A. COLLY, P. O. Box 3583, Johannesburg, Transvaal, South Africa.

LIEUTENANT CONRAD V. HAHN, Ordnance Department, U. S. A., Aberdeen Proving Ground, Maryland, and for mail, 3526 Filbert Street, Philadelphia, Pennsylvania.

MR. CHARLES EUGENE RILLIÉT, Teacher, Polytechnic High School, 509 West 1st Street, Santa Ana, California, and for mail, 4223 Brighton Avenue, Los Angeles, California.

MR. CHARLES B. SANDS, Investment Securities, 932 Chestnut Street, Philadelphia, and for mail, P. O. Box 42, Ocean City, New Jersey.

MR. FRANK W. SUTTON, Industrial Engineer, 218 Engineers' Building, Cleveland, Ohio.

MR. CHRISTOPHER VAN DEVENTER, Consulting Engineer, 438 First National Bank Building, Chicago, Illinois.

MR. LUCIEN I. YEOMANS, Industrial Engineer, 72 West Adams Street, Chicago, Illinois.

ASSOCIATE.

MR. CARL T. MACK, Electrical Engineer, H. L. Doherty Company, 60 Wall Street, New York City, New York.

MR. SYLVAN D. ROLLÉ, Draftsman, Bell Telephone Company of Pennsylvania, and for mail, 2237 North 21st Street, Philadelphia, Pennsylvania.

LIBRARY NOTES.

PURCHASES.

American Society of Heating and Ventilating Engineers. Transactions, vol. xxiv. 1918.

BACKER, H. J.—Oude chemische Werktuigen en Laboratoria van Zosimos tot Boerhaave. 1918.

Cumulative Book Index, vol. xxi. 1919.

HUBBARD, E.—The Utilization of Wood Waste. 1915.

PAUL, J. H.—Boiler Chemistry and Feed Water Supplies. 1919.

GIFTS.

Albert and J. M. Anderson Manufacturing Company, Bulletin No. 34. Boston, Massachusetts, 1919. (From the Company.)

Allis-Chalmers Manufacturing Company, Illustrated Catalogue No. 107. Milwaukee, Wisconsin, 1919. (From the Company.)

American Arch Company, Bulletin No. 3. New York City, New York, 1919. (From the Company.)

American Malleable Castings Association, Catalogue of Malleable Iron. Cleveland, Ohio, no date. (From the Association.)

American Petroleum Products Company, Booklet, "The Use of Oil as a Fuel." Cleveland, Ohio, 1919. (From the Company.)

American Well Works, Catalogue No. 149. Aurora, Illinois, 1917. (From the Works.)

Atlas Car and Manufacturing Company, Bulletin No. 1185. Cleveland, Ohio, no date. (From the Company.)

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Barrett Company, Booklet on the Methods of Analysis of the Coal-Tar Industry. New York City, New York, 1918. (From the Company.)

British Association for the Advancement of Science, Report, 1918. London, 1919. (From the Association.)

Burt Manufacturing Company, The Burt Booklet. Akron, Ohio, 1917. (From the Company.)

Canada Department of Mines, Memoirs 107, 108, 110 and 113. Ottawa, Canada, 1919. (From the Department.)

Carnegie Institute of Technology, General Information for 1919–1920. Pittsburgh, Pennsylvania, 1919. (From the Institute.)

Commercial Truck Company of America, Bulletin No. 15. Philadelphia, Pennsylvania, 1919. (From the Company.)

Connecticut Public Utilities Commission, Seventh Annual Report for 1918. Hartford, 1919. (From the Commissioner.)

Conveying Weigher Company, Catalogues Nos. A-10 and A-11. New York City, New York, 1916. (From the Company.)

Cutler-Hammer Manufacturing Company, Publications Nos. 413 and 445. Milwaukee, Wisconsin, 1919. (From the Company.)

DePere Manufacturing Company, Bulletins Nos. 1 to 7, inclusive. Chicago, Illinois, 1918–1919. (From the Company.)

Electric Service Supplies Company, Catalogue of Flood Lighting Projectors. No date. (From the Company.)

Elliott Company, Combined Catalogue. Pittsburgh, Pennsylvania, 1919. (From the Company.)

Engineer Company, Catalogue, "The Development of an Idea." New York City, New York, no date. (From the Company.)

Falls River Company, Catalogue No. 19. Kent, Ohio, 1919. (From the Company.)

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Hobart College, Catalogue, 1918–1919. Geneva, New York, 1919. (From the College.)

Horton E., & Son Company, Catalogue No. 16. Windsor Locks, Connecticut, no date. (From the Company.)

Hydraulic Press Manufacturing Company, Catalogues Nos. 43 and 80. Mount Gilead, Ohio, 1919. (From the Company.)

Illinois Electric Porcelain Company, Catalogue of Electrical Porcelain Insulations. Macomb, Illinois, no date. (From the Company.)

Iowa Railroad Commission, Fortieth Annual Report for the year ending December 3, 1917. Des Moines, 1919. (From the Commissioners.)

Iowa State College of Agriculture and Mechanic Arts, General Catalogue, 1919–1920. Ames, Iowa, 1919. (From the College.)

Jaeger Machine Company, Catalogues of Concrete Mixers. Columbus, Ohio, no date. (From the Company.)

Jewell Electrical Instrument Company, Catalogue No. 12. Chicago, Illinois, no date. (From the Company.)

Kungl. Vitterhets Historie och Antikvitets Akademien, Antikvarisk Tidskrift for Sverige. Stockholm, Sweden, 1919. (From the Academy.)

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Locomotive Stoker Company, Publication No. 15. Pittsburgh, Pennsylvania. 1916. (From the Company.)

McKiernan-Terry Drill Company, Bulletins Nos. 25 and 26. New York City, New York, 1919. (From the Company.)

Mason Regulator Company, General Catalogue No. 60. Boston, Massachusetts, 1919. (From the Company.)

Massachusetts Engineering Department, Annual Report of the City Engineer for 1919. Haverhill, 1919. (From the Department.)

Merchant & Evans Company, Catalogue of Almetl Fire Doors and Shutters. Philadelphia, Pennsylvania, 1918. (From the Company.)

Mercury Manufacturing Company, Publication, "The Trackless Train." Chicago, Illinois, no date. (From the Company.)

Metal Forms Corporation, Catalogue of Metaform Outfits. Milwaukee, Wisconsin, no date. (From the Corporation.)

National Physical Laboratory, Reports for the Years 1916–17–18. Teddington, England, 1917–1918. (From the Laboratory.)

New Hampshire Public Service Commission, Annual and Statistical Report, 1917. Manchester, New Hampshire. (From the Commission.)

New Zealand Department of Mines, Palæontological Bulletin No. 7. Wellington, 1918. (From the Department.)

The Northern Equipment Company, Catalogue of the Copes System of Feed Water Control. Erie, Pennsylvania, 1917. (From the Company.)

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Oliver Continuous Filter Company, Bulletin No. 12 of the Oliver Filters. San Francisco, California, 1919. (From the Company.)

Packard Electric Company, Bulletin No. 201. Warren, Ohio, 1919. (From the Company.)

Pennsylvania Insurance Commissioner, Forty-fifth Annual Report, part ii, 1917. Harrisburg, Pennsylvania, 1918. (From the State Librarian.)

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Pilliod Company, Booklet of the Baker Locomotive Valve Gear. Swanton, Ohio, 1918. (From the Company.)

Pittsburgh Iron and Steel Foundries Company, Catalogue of Adamite. Pittsburgh, Pennsylvania, 1917. (From the Company.)

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Schramm, Chris D. & Son, Inc., General Catalogue No. 18. Philadelphia, Pennsylvania, no date. (From Chris D. Schramm & Son, Inc.)

Seneca Falls Manufacturing Company, Inc., Catalogue No. 27, Seneca Falls, New York, no date. (From the Company.)

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Tempe Normal School, Bulletin No. 1. Tempe, Arizona, 1919. (From the School.)

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United States Shipping Board Emergency Fleet Corporation, Report on the Work of the Training of Shipyard Workers. Philadelphia, Pennsylvania, 1919. (From the Corporation.)

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University of Montana, Nineteenth Annual Catalogue for 1918–1919. Butte, 1919. (From the University.)

U. S. Light and Heat Corporation, Bulletin No. 700. Niagara Falls, New York, 1919. (From the Corporation.)

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Van Dorn Electric Tool Company, Catalogue No. 19. Cleveland, Ohio, no date. (From the Company.)

Weber Chimney Company, Catalogue No. 19. Chicago, Illinois, no date. (From the Company.)

Westinghouse Electric and Manufacturing Company, Catalogue of Electric Arc Welding. East Pittsburgh, Pennsylvania, no date. (From the Company.)

Wilmot Engineering Company, Catalogue of Breaker Machinery. Hazleton, Pennsylvania, no date. (From the Company.)

BOOK NOTICES.

HYGIENE IN MEXICO: A STUDY OF SANITARY AND EDUCATIONAL PROBLEMS. By Alberto J. Pani, C.E. Translated by Ernest L. de Gogorza. 206 pages, and contents, 12mo. New York, G. P. Putnam's Sons. Presented by the author.

General public hygiene is rather out of the scope of this JOURNAL, but it is a subject growing more and more closely related to engineering and chemistry, and besides, American interest in Mexico is so great that it will be worth while to examine the work of one who has had a large experience in the field. Senor Pani is an officer of the Carranza government and informs us in a special note that the sole purpose of the book is "to expose one of the least known, most nefarious and shameful inheritances of the past." This refers to the oppressions of the Diaz government. The entire proceeds of the sale of the Spanish edition were donated to the People's University of Mexico, which institution is especially organized to promote education among the less cultured classes of the nation.

The work is, of course, largely statistical. It gives evidence of great neglect of hygiene among the mass of the Mexican population. His investigations lead him to the opinion, expressed on page 7, that Mexico City is the most unhealthful city in the world. Some very interesting figures of the earnings and living expenses of the laborers in the city are given. One receives seventy-five cents a day (presumably U. S. money equivalent) working every day. He supports two others—mother and wife. His food costs per week, $3.62; clothing, washing and hair cutting, $.94; room rent (very squalid quarters), $.50; leaving a weekly surplus of $.19.

The book is a valuable contribution to general hygiene. Those who desire to know something of the "Mexican problem" should consult it. The work of the translator is very well done. The English is excellent, free from Spanish idioms.

HENRY LEFFMANN.

PUBLICATIONS RECEIVED.

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Ontario Bureau of Mines: Report, 1918, Sand and Gravel in Ontario, by A. Ledoux. 138 pages, illustrations, maps, 8vo. Toronto, King's Printer, 1918. Abitibi-Night Hawk Gold Area, by C. W. Knight, A. G. Burrows, P. E. Hopkins and A. L. Parsons. Larder Lake Gold Area, by P. E. Hopkins. 84 pages, illustrations, plates, maps, 8vo. Toronto, King's Printer, 1919.

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CURRENT TOPICS

Production of Glycerin from Sugar by Fermentation.—JOHN R. COFF, W. V. LINDER and G. F. BEYER (*Journal of Industrial and Engineering Chemistry*, vol. xi, pp. 842–845, 1919) have studied the production of glycerin from sugar by fermentation with yeast. Several successive sowings of the Steinberg variety of *Saccharomyces Ellipsoidens* are made, each sowing while the yeast is most active. Sodium carbonate is added at each sowing so that its concentration is from one-half to one per cent. of the weight of the nutrient solution; the yeast is thereby accustomed to an alkaline environment. The yield of glycerin is greater in an alkaline solution than in either a neutral or an acid solution. The yeast culture is added to a sugar solution, *e.g.*, a solution of black strap molasses; the most favorable concentration is from 17.5 to 20.0 grains of sugar per 100 c.c. of solution. The fermentation is carried out at a temperature of 30° to 32° C. in the presence of a small amount of ammonium chloride. Solid sodium carbonate is added to the wash from time to time; when a sufficient quantity has been added, a copious precipitate forms, evolution of gas ceases, and the yeast apparently lies dormant for a time, the precipitate eventually disappears and fermentation again occurs. The formation of a precipitate and the quiescence of the mash are essentials of the process. The best results are obtained when the total amount of sodium carbonate added is approximately 5 per cent. The entire amount cannot be added at one time, for it would permanently stop the fermentation; on the other hand, the yield of glycerin is greater the earlier the entire amount has been added in several installments with intervening cycles of precipitation, quiescent period, and fermentation. The requisite alkalinity of the medium may also be obtained by use of potassium carbonate, sodium hydroxide, potassium hydroxide or borax. When fermentation is completed, from 20 to 25 per cent. of the sugar has been converted into glycerin, the remainder into carbon dioxide and alcohol; other products, as yet unidentified are formed in small amount. The alcohol may be recovered by distillation. The glycerin is obtained by neutralizing the fermented solution with sulphuric acid, purifying by addition of coppers and lime, removing the precipitate in a filter press, concentrating the filtrate in a vacuum evaporator, and finally distilling in a suitable still. On a commercial scale, corn sugar and cane sugar were less satisfactory than molasses as a raw material. Their solutions had to be enriched with yeast nutriate of such a nature and to such an extent that these nutrients interfered with the purification of the glycerin. J. S. H.

Potash Recovery in Cement Works.—The discovery that a limited amount of soluble alkalies exists in the dust of cement plants seems to be due to Seger and Cremer, who in 1904 reported on the fact in *Thonind.-Zeit.* abstracted in *Jour. Soc. Chem. Ind.* (vol. xxiii, p. 531, 1904). The possibility of recovering this alkaline material was first pointed out by Clifford Richardson at a meeting of the American Society for Testing Materials in June, 1904. At this time Richardson stated that in a cement plant turning out 4000 barrels per day there might be a possible profit of between $100 and $200 per day. This was, of course, at the prices then ruling for potassium salts.

The data just given are taken from a report by the Department of Mines of Canada, bulletin 29, prepared by Alfred W. G. Wilson, Ph.D., Special Investigator for the War Trade Board. A considerable amount of information is given as to the results of recovery of potassium salts from the dust of the cement works, principally in the United States. Several illustrations are included, all of which are United States installations. It is also stated that potassium salts may be recovered from blast-furnace gases, and that three steel plants in the United States are experimenting on this matter, and further that in Great Britain it is believed that the yield from this source will suffice for domestic wants.

Two systems are in vogue for recovery from cement dust, the electrical precipitation under the Cottrell patents and the water spray system. A double benefit results from these methods, for not only is a valuable fertilizer obtained, of which there is a shortage in America, but a very objectionable dust is condensed. In fact, in one case it was due to legal proceedings against the works that the dust remover was installed and as a secondary result the potash yield was obtained and for a limited time, owing to the high price of potash salts, the by-product was the most profitable output of the plant.

The bulletin concludes with the advice that the problem of by-product recovery should receive the attention of all Canadian cement manufacturers. Of course, this advice will be worth considering by United States manufacturers as well. Not the least advantage will be removal of annoyance to the neighborhood by the escaping dust, especially objectionable in agriculture districts, and also very injurious to the workers in the plants. Thus we have the fortunate combination of industrial hygiene and increase of operation profit to bring about material improvement in such plants. H. L.

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